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PHYSICAL REVIEW.

MODIFICATIONS OF THE MAXWELL–RAYLEIGH AND THE ANDERSON METHODS FOR THE MEASUREMENT OF THE COEFFICIENT OF SELF-INDUCTION.

By Anthony Zeleny.

I. The Maxwell-Rayleigh Method.

THE Maxwell-Rayleigh method[1] for the measurement of the coefficient of self-induction is theoretically more free from doubtful corrections than any other known method. It has been discarded, however, on account of an experimental difficulty. The form in which the method was left by Lord Rayleigh necessitates, in addition to balancing the Wheatstone bridge and a ballistic galvanometer throw, a deflection of the galvanometer produced by the continuous current when one arm of the bridge has been overbalanced by a very small and known amount. After the introduction of this small resistance and during the interval required for the deflecting needle or coil to come to rest the resistance of the copper inductance coil is changed by heating due to the current and by variations in the temperature of the room, so that when the galvanometer reading is observed the deflecting current is produced by an overbalancing resistance which differs from the small known resistance introduced. The amount of change in the resistance of the copper coil is unknown and is usually sufficient to render results uncertain within one per cent.

The modification of the method here described renders the ob-

[1] Phil. Trans., R. S., 1882, p. 677; Henderson, Practical Electricity and Magnetism, 1898, p. 301.

jectionable measurement unnecessary and makes of it a convenient and an accurate method. In the Wheatstone bridge, Fig. 1, let

R_1 = the inductive resistance,

R_2, R_3, R_4 = the non-inductive resistances,

G = resistance of the galvanometer branch,

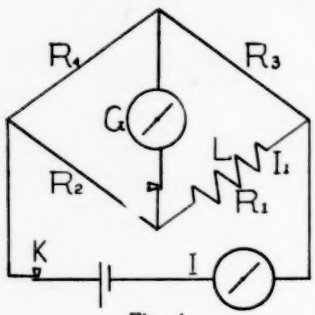

Fig. 1.

L = coefficient of self-induction of R_1,

I = current in the main circuit,

I_1 = current in the inductive coil R_1.

Also let i = induced current flowing through R_1,

q = the quantity of induced electricity passing through the galvanometer,

Q = whole quantity of induced electricity passing through the bridge on the opening of the main circuit,

R = the resistance of the Wheatstone net around the circuit with R_1, the sources of the induced electromotive force, as the origin.

When the circuit is opened by means of the key K, after a balance has been obtained, the induced electromotive force

$$e = -\frac{LdI_1}{dt}$$

and

$$L = -\frac{e}{\frac{dI_1}{dt}} = \frac{e}{\frac{I_1}{t}} = \frac{iR}{\frac{I_1}{t}} = \frac{QR}{I_1} \qquad (I)$$

Q is calculated from the measured quantity q and the known resistances. R is obtained from the known resistances, and I_1 from the measured current I and the known resistances. All of these quantities can be determined with accuracy.

For convenience of calculation the terms from which the quantities Q, R and I_1 are calculated will be substituted for these quantities in the equation (I).

$$q = \frac{(R_2 + R_4)Q}{R_2 + R_4 + G}, \quad \therefore Q = \frac{R_2 + R_4 + G}{R_2 + R_4} q = \left(1 + \frac{G}{R_2 + R_4}\right) q. \qquad (II)$$

$$R = R_1 + R_3 + \frac{(R_2 + R_4)G}{R_2 + R_4 + G} = R_1 + R_3 + \frac{1}{\frac{1}{G}\left(1 + \frac{G}{R_2 + R_4}\right)}. \quad \text{(III)}$$

$$I_1 = \frac{(R_3 + R_4)I}{R_1 + R_2 + R_3 + R_4} = \frac{I}{1 + \frac{R_1 + R_2}{R_3 + R_4}} = \frac{I}{1 + \frac{R_2}{R_4}}. \quad \text{(IV)}$$

$\left(\frac{R_1 + R_2}{R_3 + R_4} = \frac{R_2}{R_4}\right.$ because when the bridge is balanced $\left.\frac{R_2}{R_4} = \frac{R_1}{R_3}\right)$

Substituting these values in equation (I),

$$L = \frac{QR}{I_1} = \frac{q}{I}\left(1 + \frac{R_2}{R_4}\right)\left[\left(1 + \frac{G}{R_2 + R_4}\right)(R_1 + R_3) + G\right] \quad \text{(V)}$$

When the quantity q is measured by comparison with the free charge[1] of a standard condenser,

$$q : CE :: d : D$$

and

$$q = \frac{CE}{D}d, \quad \text{(VI)}$$

where d is the throw due to q and D the throw due to the quantity CE.

Substituting this value of q in equation (V),

$$L = \frac{CE}{D}\cdot\frac{d}{I}\left(1 + \frac{R_2}{R_4}\right)\left[\left(1 + \frac{G}{R_2 + R_4}\right)(R_1 + R_3) + G\right]. \quad \text{(VII)}$$

When the quantity q is measured by comparison with the quantity induced in the secondary circuit of a standard mutual inductance coil[2] whose coefficient of self-induction is M, the quantity of induced electricity, $Q = MI_P/R_S$

$$\therefore q : \frac{MI_P}{R_S} :: d : D$$

and

$$q = \frac{MI_P}{R_S D}d$$

and $L = \frac{MI_P}{R_S D}\cdot\frac{d}{I}\left(1 + \frac{R_2}{R_4}\right)\left[\left(1 + \frac{G}{R_2 + R_4}\right)(R_1 + R_3) + G\right]$. (VIII)

When $R_1 = R_3$ and $R_2 = R_4$, as is usually the case, equation (VII) becomes

[1] A. Zeleny, The Capacity of Mica Condensers, PHYS. REV., 1906, Vol. 22, p. 65.

[2] A. Zeleny, Galvanometer Measurements, PHYS. REV., 1906, Vol. 23, p. 399.

$$\mathbf{L = 2\frac{CE}{D}\cdot\frac{d}{I}\left[2R_3\left(1+\frac{G}{2R_2}\right)+G\right]} \qquad \text{(IX)}$$

and equation (VIII) becomes

$$L = 2\frac{MI_P}{R_3 D}\cdot\frac{d}{I}\left[2R_3\left(1+\frac{G}{2R_2}\right)+G\right]. \qquad \text{(X)}$$

For simplifying the calculations, G can be made some exact multiple of R_2 by introducing the proper resistance into the galvanometer circuit.

ABSOLUTE MEASUREMENTS.

The coefficient of self-induction may be determined by means of any one of the equations (VII), (VIII), (IX) or (X). The equation (IX), however, is to be preferred ordinarily on account of its simplicity and the ease with which the necessary measurements are made. The limit of attainable accuracy is that of direct deflection methods, *i. e.*, about .01 per cent., but may be somewhat increased by use of the differential galvanometer. The error due to the small self-inductance in the non-inductive arms of the bridge and to the electrostatic capacity of all of the four arms is very small and should become appreciable only in the measurement of very small inductances. The error introduced, however, is much smaller than with any of the alternating current methods.

COMPARISON MEASUREMENTS.

This method may be employed to compare coefficients of self-induction with a standard self-inductance. The three non-inductive resistances and the galvanometer resistance are maintained constant, and, in the fourth arm of the bridge, a variable non-inductive resistance is placed in series first with the standard inductance coil and then with the one of unknown inductance. This non-inductive resistance is varied in obtaining the balance first for the standard and then for the unknown inductance. The current may be varied, if desirable, so that approximately equal throws are obtained in the two cases. In the expressions for the values of the coefficients as obtained from equation (VII) all of the quantities except d and I are common to both, so that

$$L : L_1 :: \frac{d}{I} : \frac{d_1}{I_1}$$

and

$$L_1 = \frac{d_1 I}{d I_1} L. \qquad \text{(XI)}$$

But if the current is left unchanged

$$L_1 = \frac{d_1}{d} L \qquad \text{(XII)}$$

and the only observations necessary are the throws given by the known and unknown inductances. If the electromotive force is constant the current need not be observed, because when a balance is obtained the total resistance in the circuit is the same in the two cases.

Experimental Results.

In a series of measurements by this method the apparatus employed was arranged as shown in diagram in Fig. 2. R_2, R_3 and R_4 represent three arms of the Wheatstone bridge. In the fourth arm are the inductive resistance whose coefficient of self-induction

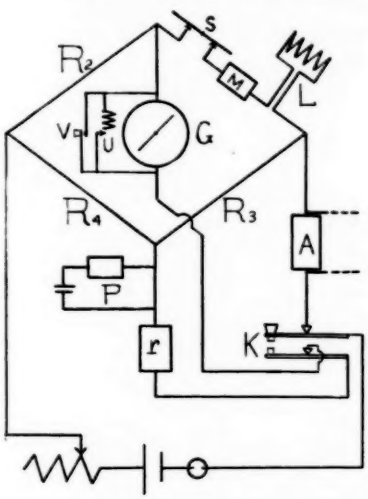

Fig. 2.

L is to be measured, the adjusting resistance M, and the sliding resistance S which is described in detail later. U represents the shunt employed during the adjustment of the balance and V the

short-circuiting key used in bringing the coil to rest so as not to pass beyond the null point. P is a secondary circuit for compensating any thermo-electric current that may exist in the galvanometer circuit.[1] The resistance G in the galvanometer circuit can be varied by means of the resistance r. A is a standard resistance employed for measuring the current in the main circuit, the difference of potential between its binding posts being determined by a Wolff potentiometer. By means of a single tap, the key K opens the main circuit and after all the measurable part of the induced electricity has passed through the Wheatstone net, but before the galvanometer coil has turned an appreciable distance, it opens the galvanometer circuit. The quantity of electricity passing through the galvanometer can then be compared directly with the quantity discharged from a standard condenser. (See former papers of the writer, referred to above.)

A proper construction of the sliding resistance S is essential for the convenience and success of this method. The contacts must be

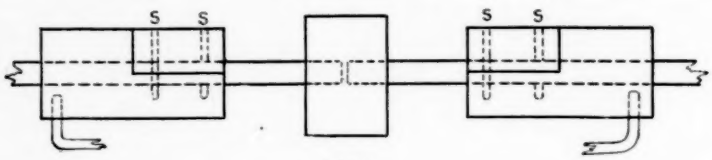

Fig. 3.

good and the slide easily moved so that, if necessary, the operator can give it a continuous slow motion and thereby compensate for the gradual change of resistance due to the alteration of the temperature of the inductance coil. The sliding resistance employed is shown in Fig. 3. It is composed of two wires, each 35 cm. long, one being a No. 16 copper and the other being a No. 16 manganin wire. The resistance of the manganin wire is a little more than .05 ohm, so that with 0.1 ohm as the smallest resistance in the adjusting box, M, a perfect balance can be obtained by first adjusting the resistance in the box, then the slide on the manganin wire, and finally, for the most accurate adjustment, the slide on the copper wire. Each slide consists of a block of copper 3.5 cm. long, with a hole through its

[1] A Leeds and Northrup H type moving coil galvanometer of 127 ohms resistance was employed.

center of the same diameter as that of the wire. A part of each slide is cut into two pieces, as shown in the figure, and the two parts are held together by a strong rubber band (not shown) and kept from sliding sidewise by the pins *SS*. The face of the smaller section was filed, so as to always make a contact at the end of the block. This makes a satisfactory slide, free from contacts having strong thermo-electric power. It is necessary, however, to polish the wires with fine emery cloth before using. This is especially true of the copper wire; a small quantity of oxide making the resistance of the contact variable so that no satisfactory adjustment is possible.

After a balance of the bridge was obtained the thermo-electric current in the galvanometer circuit was compensated and then again the final balance of the bridge made. The quantity of induced electricity passing through the galvanometer at the opening of the main circuit was compared with the quantity discharged from a standard condenser. The resistance of the galvanometer circuit was measured after each set of observations.

The coefficients for two coils wound on the same spool were determined; and to test the accuracy of the values obtained, the coefficients of the two placed in series with the currents first in the same and then in opposite directions were determined as was also the coefficient of mutual induction.

The following observations were common to all the self-induction determinations:

$t = 20°.2$ to $20°.3$ C.

$R_2 = R_4 = 99.99$ ohms.

$A = 9.9966$ ohms.

$R_3 = 40.000$ except in case of coil No. 2, where $R_3 = 10.014$ ohms.

E.M.F. of standard cell on potentiometer = 1.01869 (German) volts.

E.M.F. of standard cell used for charging condenser = 1.01883 (German) volts.

$C = 1.00136 \times 10^{-6}$ farads.

Distance of straight scale = 127.2 cm.

The remaining data are given in Table I.

TABLE I.

Coils.	Induction Throws (d).	Condenser Throws (D).	Potentiometer Reading-Volts.	R of Galv. Circuit.	L in Henrys.
No. 1 + No. 2	19.421 ±3	19.908 ±2	0.50906 ±3	1686.0 ±1	.095391
No. 1 − No. 2	19.471	19.893	0.50895	393.75	.024766
No. 1	19.535	19.824	0.50877	865.82	.051049
No. 2	19.216	19.445	0.93644	363.65	.009042
Mutual inductance of No. 1 and No. 2.	19.730	19.700	0.69205	Resistance of secondary circuit. 1196.12.	M in Henrys .017653

The lower part of Table I contains the observations necessary, in addition to the above data, for the determination of the coefficient of mutual induction of the two coils. The method used was that employing the relation $M = QR_s/I_p$, the main and secondary circuits being opened in succession as in the self-induction determinations. The apparatus was arranged as shown in diagram in Fig. 4, the parts serving a similar purpose as those in Fig. 2 are represented by similar characters, while C represents the two inductance coils.

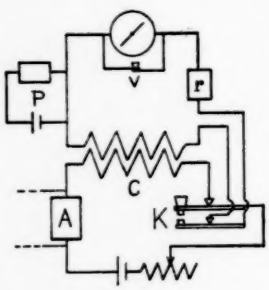

Fig. 4.

The galvanometer throws given are the average of five or more, varying among themselves by an amount rarely exceeding .01 cm., and all have been changed from the observed straight scale to the equivalent circular scale readings.

The coefficients of the two coils when in series with the current in the same direction (L_{1+2}) and with the current in opposite directions (L_{1-2}) are given by the equations:

$$L_{1+2} = L_1 + L_2 + 2M,$$
$$L_{1-2} = L_1 + L_2 - 2M,$$

where M is the coefficient of mutual induction. Hence

$$L_{1+2} + L_{1-2} = 2(L_1 + L_2) \qquad \text{(XIII)}$$

and

$$\frac{L_{1+2} - L_{1-2}}{4} = M. \qquad \text{(XIV)}$$

Substituting the determined values as given in Table I:

In equation (XIII)

$$2(L_1 + L_2) = .120182$$
$$L_{1+2} + L_{1-2} = .120157$$
$$\text{Difference} = .000025 = .021 \text{ per cent.}$$

In equation (XIV)

$$M = \frac{L_{1+2} - L_{1-2}}{4} = .017656$$

The independently determined value of M $= .017653$

$$\text{Difference} = .000003 = .017 \text{ per cent.}$$

This test shows that the results obtained are correct within the limits of observational error as the greater of the two differences, *i. e.*, .021 per cent. would be introduced by a constant error of less than one-half a tenth of a millimeter in the observed throws. It also shows that in the employment of this method the error due to the self-induction in the non-inductive resistances[1] and to their electrostatic capacities must be very small.

II. The Anderson Method.

The Anderson method and its modifications have been accepted by many as the most satisfactory of all the known methods. For this reason, a comparison of the results obtained by the above modified Maxwell-Rayleigh method was made with the results obtained by this one which, however, it was found necessary to modify as described below.

A Modification of the Original[2] *Null Method.* — In the original Anderson method as it is now commonly employed the condenser remains permanently connected in the circuit so that, after a balance is obtained and the circuit is opened, the liberated absorbed charge as well as the free charge of the condenser affects the galvanometer. For this reason the results obtained are dependent upon the period of the galvanometer and upon the resistance in the condenser circuit, and may be in error by more than one per cent. even when a good condenser is employed. No accurate determinations can be made in this way unless the effective capacity of the condenser for the

[1] A Leeds and Northrup box bridge with coils wound on wooden spools was employed.

[2] A. Anderson, Phil. Mag., 1891, XXXI., p. 329.

particular apparatus is first obtained, but this is not practicable. The capacity values of standard condensers are usually determined by standardizing bureaus by an intermittent current method giving 100 interruptions per second. This should give the "free charge" capacity values of good condensers with sufficient accuracy for all ordinary purposes. The rated values can therefore be employed when using the condensers as described in a previous paper.[1]

The determinations by this method were made therefore by using the discharge key shown in Figs. 2 and 4, the main and the galvanometer circuits being opened in succession, as in the cases previously described. The rated capacity value of the condenser was then the effective value. The results obtained were

by the Modified Maxwell-Rayleigh Method $L = .095391 \pm 9$

and by the Modified Anderson Method $L = .095386 \pm 9$,

an agreement well within the limits of observational error. For the same degree of sensitiveness, however, the Anderson method required a current strength three times that employed in the other method.

The Secohmmeter Method. — The value of the coefficient for the same coil was obtained by using the Anderson method modified[2] by alternating the current by means of a secohmmeter. The object was to compare the results with those obtained with the other methods and to observe the variations with the change of speed of the secohmmeter. The electrostatic effects due to the commutator were eliminated in the usual manner.[3] The currents in the various branches did not effect the inductance in the coil, for when a balance was obtained no appreciable change was observed on reversing the coil, which was placed about a meter from the other parts of the apparatus.

The chief difficulty experienced was from the thermo-electric current due to the unequal warming, by friction, of the brushes on the commutator. To correct for this, the sechommeter was brought to rest after a preliminary balance was obtained and the null point taken as affected by the thermo-electric current. The final balance was then made using the new null point. The magnitude of this correction affected the results by .04 per cent. at the lowest and by

[1] A. Zeleny, loc. cit.

[2] Fleming and Clinton, Phil. Mag., 1903, V., p. 493.

[3] The Electrician (London), 1897, p. 379.

.005 per cent. at the highest speed employed. The thermo-electric current, however, was somewhat irregular so that at the lower velocities an error of .01 per cent. due to this cause is possible. The results obtained are given in Table II.

TABLE II.

No. of Alternations per Second.	L	Increment $\times 10^6$.	No. of Alternations per Second.	L	Increment $\times 10^6$.
0	.095386	—	34.6	.095559	173
10.16	.095442	56	42.6	.095565	179
15.62	.095499	113	50.0	.095581	195
27.6	.095545	159	57.6	.095590	204

For the same current flowing through the main circuit this method for the higher speeds was about 200 times as sensitive as the same method when employed without the secohmmeter, but on account of continuous variations of the balance the actual working sensibility was only about ten times as great. It appears from Table II., that notwithstanding the great sensitiveness of this method it cannot be employed for measurements of the highest accuracy without the application of doubtful corrections of considerable magnitude.

These variations of the values are due principally to the effect of the inductances in the non-inductive branches and to the electrostatic capacity in all the branches, the effect varying with the rate of the alternations. This subject is ably treated by Rosa and Grover.[1] The variations are no doubt due partly also to the varying effect of the absorbed charges in the condenser.

SUMMARY.

The experimental difficulty of the Maxwell-Rayleigh method for measuring the coefficient of self-induction is avoided by omitting the measurement with the small known resistance for disturbing the balance in the Wheatstone net and substituting for it the measurement of the current in the main circuit. The modified method presents for work of precision an advantage over all other known methods on account of a greater freedom from uncertain corrections.

[1] Bulletin of the Bureau of Standards, 1905, Vol. I., p. 304.

The method as expressed by equation (1) is also valuable from an educational standpoint on account of the simplicity of the mathematical relations involved.

A simple modification enables the comparison of inductances to be made with ease.

The Anderson null method as modified by the writer so as to render only the free charge of the condenser effective is shown, experimentally, to be capable of as high a degree of accuracy as the modified Maxwell-Rayleigh method. It possesses a slight disadvantage from a theoretical standpoint, is less sensitive, and requires a double balancing; but has the advantage of being a null method and of not requiring the measurement of several electrical quantities necessary in the employment of the other method.

The modification of the Anderson method by the employment of a secohmmeter gives results varying appreciably with the rate of the alternations of the current.

PHYSICAL LABORATORY,
THE UNIVERSITY OF MINNESOTA,
December 22, 1906.

THE ABSORPTION OF HYDROGEN BY METAL FILMS.

By Walter Heald.

IT is known that under certain conditions a number of the metals absorb gas. Some, such as silver and gold, are capable when in a molten state of absorbing it from the air, as is the case with other liquids, this gas being given up again when the metal solidifies. Palladium absorbs very large quantities of hydrogen even when in the solid state. For instance, if a piece of palladium is used as the negative pole in the electrolysis of a dilute solution of sulphuric acid, it takes up about nine hundred times its own volume of hydrogen.

Skinner [1] has found that most of the metals give off hydrogen when used as cathode in a discharge tube, which shows that in their natural condition the metals contain a certain amount of absorbed hydrogen.

In the present investigation, while trying to determine the quantity of gas that a given amount of metal evolved when vaporized by a large electric current in an atmosphere of hydrogen, it was noticed that after the vaporization took place, the gas pressure gradually decreased. This could not be accounted for by a decrease in temperature, which could last only a short time, because the pressure continued to decrease in some cases for an hour or more. Before the electric current had passed the gas pressure was noted as constant. After it had been broken, apparently the only change that had taken place in the tube was that a metal film had been deposited on the walls of the tube. Therefore, the conclusion is drawn that the newly deposited metal gradually absorbed hydrogen from the molecular state.

In the case of its absorption by a palladium cathode during electrolysis of a dilute solution of sulphuric acid, the hydrogen is in a nascent state. Such may also be the case when it is absorbed

[1] Phys. Rev., vol. 21, no. 1, July, 1905.

by hot metals, the temperature at least aiding dissociation. If, then, hydrogen, which is in the molecular state is absorbed by a cold metal film, it seems most likely that the metal in this condition dissociates the hydrogen, though possibly it has a strong affinity for and absorbs the hydrogen molecule. At any rate it is interesting to learn something concerning the absorption of hydrogen by films of different metals produced in the above manner.

The purpose of this investigation was therefore to obtain deposits of a number of different metals in a tube containing hydrogen, and to measure the amount and rate with which each absorbed the gas. The deposits were produced as indicated by vaporizing the metal in a discharge tube by means of a large electric current and allowing it to condense on the walls of the containing tube. It was hoped that the quantity of the active metal might also be determined but this proved too small to measure.

APPARATUS AND METHOD OF EXPERIMENT.

The form of discharge tube used is shown by diagram in Fig. 1. The brass rods *R* are sealed with de Khotinsky's laboratory cement in the glass tubes *P* which fit in the ground glass joints *J*. Electrodes *E* represent the metal to be tested. These were either screwed or clamped in the ends of the brass rods. Glass tubing enclosed the brass rods so as to confine the discharge to the metals used as electrodes.

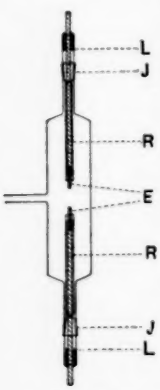

Fig. 1.

The discharge tube was connected to an air pump and a tube containing P_2O_5 for drying purposes. It was also connected to a flask containing dry hydrogen, which was generated by electrolysis of dilute phosphoric acid, and to a McLeod gage used for measuring pressure. This gage magnified the pressure one hundred times.

After exhausting the discharge tube it was allowed to stand in connection with the P_2O_5 tube several hours before admitting hydrogen.

The current used for vaporizing the metal was supplied by a battery of small accumulators. Usually a potential difference of about a thousand volts was employed, the amount of current being regu-

lated by placing a copper fuse wire in the circuit. The current passed until this melted, which took from about a half second to two or three seconds. Most of the time the fuse wire used was No. 32 *B* and *S* gage. In some cases a No. 36 wire was placed in parallel with it.

The discharge tube was filled with hydrogen to a pressure of from one to six millimeters and through this the vaporizing current was passed. The volume of the chamber occupied by the gas was about five hundred cubic centimeters.

With most of the metals there was considerable difficulty in obtaining a sufficient deposit to give appreciable results. Two methods for vaporizing the metal were tried. The first was to use two pieces as electrodes, these being placed about an inch apart. The second was to connect the two brass rods *R* with a wire of the metal to be tested; then to melt it by sending an electric current through it, so that the arc, arising when the connection was broken by fusion, vaporized part of it. The first method proving the more satisfactory was the one used most of the time. Following is given an account of metals tested and results obtained.

Cadmium.

Cadmium was tested first because of its having a comparatively low boiling point, hence would vaporize easily. With a cadmium disk serving as cathode and a steel disk as anode, the metal film seemed to come wholly from the cathode, since the deposit formed only on the walls of the tube near the cathode. Besides, the surface of the cathode alone should erosion. The same was true with other metals. Therefore it is assumed that the deposit in all cases was of the same material as the cathode.

Three deposits of cadmium were produced each accompanied by some absorption of gas. In the case where it was heaviest the following results were obtained.

Time deposit was made 2:10 P. M.

Time, P. M.	Pressure, mm.
2–10–30	1.365
2–15–30	1.340
2–25–30	1.330

After this the pressure remained constant.

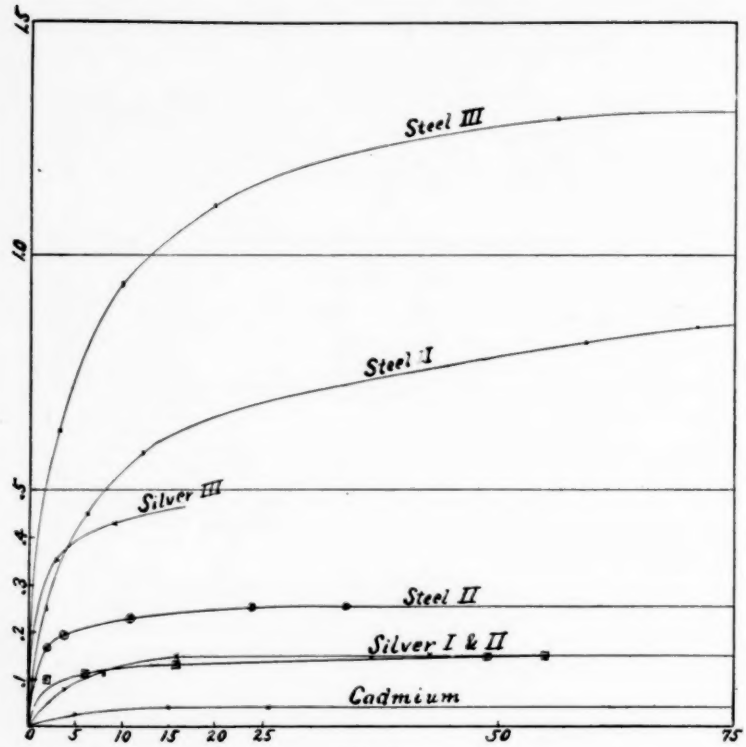

Fig. 2.

The absorption curve plotted from these results is found in Fig. 2 in which the ordinates represent the decrease in gas pressure.

STEEL.

Both methods were tried for a depositing of steel. By the first, with disk electrodes, the following results were obtained:

1. A fairly heavy deposit over a length of three or four centimeters of the tube:

Time deposit was made 10–13–30 A. M.

Time, A. M.	Pressure, mm.
10–14	4.48
10–16	4.23
10–20	4.03
10–26	3.90
11–13	3.67
11–25	3.638
11–58	3.629
P. M.	
2:30	3.628

2. Only a very light deposit:
Time of deposit 8:58 A. M.

Time, A. M.	Pressure, mm.
8-58-10	2.61
9-00	2.44
9-02	2.42
9-09	2.38
9-22	2.355
9-32	2.36

3. The deposit was heavier than either of the other two:
Time of deposit 9:35 A. M.

Time, A. M.	Pressure, mm.
9-35-10	5.626
9-38	5.0
9-45	4.687
9-55	4.515
10-37	4.336
11-37	4.285
P. M.	
1-38	4.302

The absorption curves plotted from these results are also in Fig. 2. Obviously the difference in the curves arises from the quantity of metal deposited.

Using the second method, there was only one case in which any appreciable deposit was obtained, and then not only was the steel wire vaporized but also part of the ends of the brass rods and some of the solder used to hold the clamps; hence results are not given.

Silver.

A silver disk was used as cathode, a steel disk as anode. Only slight deposits were obtained, except in case 3 in which it was fairly heavy. Following are the results:

1. Time of deposit 9–8 A. M.

Time, A. M.	Pressure, mm.
9-8-10	2.162
9-12	2.08
9-16	2.05
9-24	2.01
9-45	2.015
9-51	2.01

2. Time of deposit 10–35–30 A. M.

Time, A. M.	Pressure, mm.
10–35–40	2.09
10–37–30	1.987
10–41–30	1.976
10–51–30	1.954
11–24–30	1.94
11–35	1.942

3. Time of deposit 11–40 A. M.

Time, A. M.	Pressure, mm.
11–40–5	2.74
11–43	2.39
11–49	2.308

See Fig. 2 for absorption curves plotted from these results.

ZINC.

When making experiments with zinc the first method only was used with electrodes of zinc wire. The results seemed to be rather inconsistent. Part of the time there was apparently a slight increase of pressure and part of the time a slight decrease after the zinc film had been deposited. The effect, however, was in each case small enough to be accounted for by error in reading the pressure.

ALUMINIUM.

Several attempts were made to obtain deposits of aluminium, using pieces of wire for both anode and cathode. In two cases a slight one was obtained. There was very little if any decrease in pressure with time. There was, however, quite a decrease arising from the momentary current. In the first case this was .42 mm. and in the second .17 mm. It is probable therefore that all absorption took place within five seconds after the deposit was made.

PLATINUM.

Using small platinum wires as electrodes, slight deposits were obtained. As with aluminium there seemed to be no decrease in pressure after the deposition took place, but quite a large decrease during the passage of the current. In one instance this amounted to .53 mm. Since it is known that platinum absorbs large quantities of hydrogen, and very rapidly under certain conditions, it may

be that with the platinum film as with the aluminium, the gas was absorbed at such a rapid rate as to fully charge the metal before the pressure could be read.

Of the metals studied, cadmium, silver and steel gave definite results as to the rate of absorption of hydrogen by the films ; aluminium and platinum merely indicate that marked absorption occurs on depositing ; while zinc is the only metal tested which did not exhibit this quality.

I wish to express here my appreciation and thanks to Dr. Skinner for his help and inspiration to me in this work, as well as for the laboratory facilities placed at my disposal at the University of Nebraska where the work was done.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

ON THE SUSCEPTIBILITY OF MIXTURES OF SALT SOLUTIONS.[1]

By J. C. McLennan and C. S. Wright.

IN the following paper an account is given of some measurements made on the magnetic susceptibility of solutions of manganese, aluminium and copper sulphates in water, and several mixtures of these with the object of obtaining information which might be of service in explaining the behavior of the magnetic alloys recently discovered by Heusler,[2] and more recently studied by Gumlich,[3] Austin,[4] Fleming and Hadfield,[5] Take,[6] Hill[7] and Guthe and Austin.[8]

The method followed in measuring the susceptibility of the solutions was that suggested by Lord Kelvin,[9] and afterward used by Wills[10] in his experiments on the susceptibility of weakly magnetic solids and liquids, and later by Burton and Phillips[11] in their investigation of the magnetic properties of iron in the colloidal state.

The arrangement of the apparatus is indicated in Figs. 1 and 2. A rectangular slab *S*, Fig. 1, of the substance whose susceptibility is to be studied was suspended from one arm of a balance *B* with its long axis vertical, and its lower face in the strongest part of the field midway between the poles of an electromagnet.

[1] Read before the American Physical Society at the December meeting in New York, 1906.

[2] Fr. Heusler, W. Starck, and E. Haupt, Verh. der Phys. Ges., 5, p. 219, 1903. Heusler, Starck, and Haupt ueber der ferromagnetischen Augenschaften von Legierung unmagnetischer Metalle, Marburg, 1904.

[3] Gumlich, Annalen der Physik, 16, p. 535, 1905.

[4] Austin, Verh. der Deut. Phys. Gesell., 6, p. 211, 1904.

[5] Fleming and Hadfield, Proc. Roy. Soc., 76, p. 271, 1905.

[6] R. Take, Verh. der Deut. Phys. Gesell., 7, p. 133, 1905. Take, Ann. der Physik, Vierte, Folge, Band 20, p. 849.

[7] Hill, PHYS. REV., 21, 335, 1905; 23, No. 6, p. 498, 1906.

[8] Guthe and Austin, Bulletin of the Bureau of Standards, Washington, Vol. II., No. 2, August, 1906, p. 297.

[9] Brit. Assoc. Rep., 1890, p. 745.

[10] Wills, PHYS. REV., April, 1898; ib., March, 1905.

[11] Burton and Phillips, Proc. Camb. Phil. Soc., Vol. XIII., Pt. V.

If k = the susceptibility of the material of the slab.

A = the area of the horizontal base of the slab.

H = the strength of the magnetic field at the point midway between the poles.

H_1 = the strength of the magnetic field at the upper surface of the slab.

Then the pull P in grams due to the magnetic field is given by

$$P = \frac{kA(H^2 - H_1^2)}{2g} \text{ grams.}$$

The electromagnet used in the investigation was provided with prismatic pole pieces with plane vertical faces 5 cm. wide, and about 1.5 cm. in height. These were adjusted to approximately 1.5 cm. apart, and kept in one fixed position throughout the different measurements. In determining k for a selected liquid the slab S was

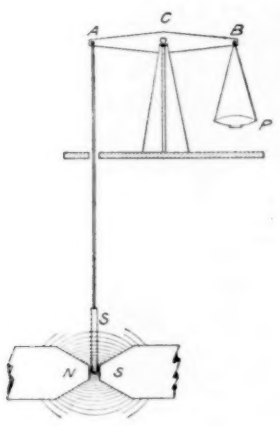

Fig. 1.

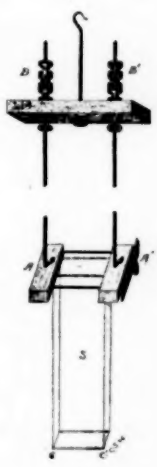

Fig. 2.

replaced by a deep rectangular glass vessel, Fig. 2, which measured internally 8 cm. in height, .78 cm. in width and 2.44 cm. in breadth. Preliminary measurements were first made on the pull exerted on the empty cell when currents of different intensities were passed through the electromagnet. The solution to be examined was then placed in a glass cell, and the pull on the filled cell again ascertained for different strengths of field. By combin-

ing these two sets of measurements the pull in the selected solution itself was obtained and the susceptibility calculated for the different strengths of field from the formula

$$k = \frac{2fg}{A(H^2 - H_1^2)}.$$

In measuring the intensity of the magnetic fields used in the experiments, a slab of wood whose dimensions were approximately the same as the internal dimensions of the glass cell was inserted in the frame in place of the cell. To the narrow vertical and lower sides of this wooden slab a strip of tinfoil was pasted, and connection was made by contact pieces at A and A', Fig. 2, to the supporting rods. Wires were attached to these at B and B', and through them currents of different strengths sent to the tinfoil circuit.

If the current carried by the tinfoil circuit be i amperes, and the length of the horizontal portion of the foil be l cm., then the pull P in grams exerted by a field H on the lower strip of the foil can readily be shown to be given by the relation

$$P = \frac{Hli}{10g}$$

and consequently the strength of the field by the equation

$$H = \frac{10Pg}{li}.$$

The use of this method permitted the field strengths to be determined with accuracy and also with considerable ease. In all the experiments the current strengths were measured by Weston instruments which previously had been standardized.

I. Determination of Magnetic Field.

As wood itself is slightly diamagnetic the different magnetic fields were found to exert a force on the slab even when no current was passing through the foil. It was therefore necessary to allow for this effect in calculating the field strengths. A set of measurements on the different fields used is given in Table I.

TABLE I.

Magnetic Field.

i in E.M.	i in Foil.	Pull on Slab no i in Foil.	Pull on Slab with i in Foil.	Pull on Foil.	$H = 10Pg/li$. $l = 3.025$ $g = 980.5$.
6.25	.13	−.0094	.1697	.1791	4465
6.4	—	−.0098	—	—	—
6.8	.13	−.0106	.1783	.1889	4710
7.15	.13	−.0113	.1837	.1950	4862
7.4	.13	−.0115	.1899	.2014	5022
7.95	.13	−.0132	.1959	.2091	5214
8.73	.13	−.0149	.2068	.2217	5528
9.05	—	−.0157	—	—	—
9.4	.13	−.0164	.2153	.2317	5777
10.0	.13	−.0177	.2235	.2412	6014
10.25	—	−.0185	—	—	—
10.6	.13	−.0190	.2297	.2487	6201
10.95	.13	−.0198	.2337	.2535	6321
11.41	.13	−.0204	.2363	.2567	6500
11.55	—	−.0209	—	—	—
12.15	.13	−.0224	.2466	.2690	6707
12.95	.13	−.0244	.2553	.2797	6974
14.05	—	−.0268	—	—	—
14.10	.13	−.0269	.2687	.2956	7367

In column I. are given the strengths of the current in amperes in the electromagnet, in column II. the strengths of the current in amperes in the foil circuit, in column III. the pulls on the wooden slab when there was no current in the foil circuit, and in column IV. the pulls on the slab, with the current passing through the foil circuit. Columns V. and VI. contain respectively the deduced values of the pulls on the foil circuit and the corresponding calculated values of the magnetic field strengths.

TABLE II.

Strength of Upper Part of Field.

i in E.M.	i in Foil.	Pull on Slab no i in Foil.	Pull on Slab with i in Foil.	Pull Due to i in Foil.	H (Calc.). $H = 10Pg/li$.
5.85	.1	−.0085	−.0008	.0077	248
9.0	.1	−.0155	−.0062	.0093	302
10.27	.1	−.0183	−.0088	.0095	308
12.6	.1	−.0235	−.0127	.0108	340
14.7	.1	−.0280	−.0164	.0116	376

A second set of measurements was made with the slab reversed, so that the current in the foil circuit passed over the upper surface of the slab instead of along the lower one, as in the previous measurements. A set of these weighings is given in Table II., and from them the value of H_1 deduced. These values, it will be seen, are small compared with the corresponding one for H, and are therefore negligible compared with the latter.

II. SUSCEPTIBILITY OF WATER.

As a control experiment a preliminary set of measurements was made upon the susceptibility of water, and the mean of a number of results found with fields ranging from 4,000 to 8,000 C.G.S. units, gave -7.33×10^{-7}, a value which is in good agreement with the results of other investigators.[1]

III. SUSCEPTIBILITIES OF SALT SOLUTIONS.

Manganese Sulphate, $MnSO_4$. — In experimenting with this salt a standard solution was prepared which was found by a gravimetric analysis to contain .0792 gram of manganese per c.c. The susceptibility was determined for fields ranging from 4,700 to 7,315 C.G.S. units, and the results of the different weighings are recorded in Table III.

[1] Susceptibility of water from a Table given by H. D. Stearns, PHYS. REV., Vol. 16, pp. 1–10, January, 1903.

	T	$k \times 10^7$	Reference.
1885 Quincke.	20°C.	−8.4	Wied. Ann., Vol. 35, p. 137, 1888.
1888 DuBois.	15 C.	−8.6	Wied. Ann., Vol. 35, p. 137, 1888.
1892 Henrichsen.		−7.5	Wied. Ann., Vol. 45, p. 38, 1892.
1895 Curie.	18 C.	−7.9	Jour. de Phys., p. 106, 1895.
1896 Townsend.	10 C.	−7.7	Proc. Roy. Soc., London, 60, p. 186, 1898.
1898–1901 Koenigsberger.	20 C.	−8	Wied. Ann., Vol. 66, p. 698, 1898, and Drude's Ann., Vol. 6, p. 506, 1901.
1898 Fleming & Dewar.		−7.4	Proc. Roy. Soc., 63, p. 328, 1898.
1899 Jaeger & H. Meyer.	18 C.	−6.6	Wied. Ann., Vol. 67, p. 712, 1899. Drude's Ann., Vol. 6, p. 870, 1901.
1903 H. D. Stearns.	22 C.	−7.33	Phys. Rev., Vol. 16, pp. 1–10, 1903.
1904 A. P. Wills.		−7.22	Phys. Rev., Vol. 22, p. 188, 1905.

TABLE III.

Solution $MNSO_4$. .0792 gm. per c.c.

H	Field Pull on Cell Water.	Pull on Cell Solution.	Pull on Salt in Solution.	$K=\frac{2Pg}{AH^2}$, $10^7\times K$, $A=1$ g sq. cm., $g=980.5$ dynes.
4,700	−.0267	.4448	.4715	213.38
4,955	−.0296	.4953	.5249	213.82
5,250	−.0334	.5545	.5877	213.26
5,500	−.0370	.6173	.6543	216.32
5,550	−.0375	.6245	.6618	214.87
6,000	−.0445	.7383	.7828	217.47
7,315	−.0638	1.0833	1.1471	214.40
				Mean 214.79

If we define molecular susceptibitity, M_s, by the equation

$$M_s = \frac{MK}{d}$$

where M is the molecular weight of the salt, K the susceptibility of the solution, and d the number of grams of the salt per c.c. in solution, we obtain from the above table the value .014915 as the molecular susceptibility of the salt.

In order to test whether the molecular susceptibility depended on the concentration, the standard solution was diluted and the susceptibility found for different concentrations. The results which are given in Table IV., show that the molecular susceptibility did not vary appreciably with the concentration.

Copper Sulphate. — Similar measurements made on a solution of

TABLE IV.

$MnSO_4$. Relation of Concentration to Susceptibility.

Concentration Grams of Mn per c.c.	Susceptibility of Solution 10^5K.	Molecular Susceptibility M_s.
.0792	214.7	.014925
.0264	74.51	.01552
.01582	41.28	.01433
.0088	23.78	.01486
		Mean M_s = .01491

copper sulphate containing .0191 gram of copper per c.c., gave a molecular susceptibility of + .00153.

Aluminium Salts. — When the susceptibility of a solution of aluminium sulphate containing .018 gram of metal per c.c. was measured, it was found to give a small negative value showing that this salt in solution is slightly diamagnetic. On account of the fact that aluminium is generally given as being weakly paramagnetic, and that no results seem to have been recorded for the salts of this metal, the susceptibilities of aluminium nitrate and of aluminium chloride were also determined and the results obtained are given in Table V.

TABLE V.

Salt.	Concentration gms. of Metal per c.c.	Susceptibility of Solution $10^7 K$.	Molecular Susceptibility M_s.
Aluminium Sulphate $Al_2(SO_4)_3$	.018	− .64	−.00018
Aluminium Nitrate $Al_2(NO_3)_6$	.097	+ .33	+.00002
Aluminium Chloride Al_2Cl_6	.094	−1.04	−.00005

From this table it will be seen that while the sulphate and the chloride are diamagnetic, the nitrate is paramagnetic. This difference between the susceptibilities of the metal and some of its compounds forms a parallel to the case of copper, where the pure metal is diamagnetic at ordinary temperatures while the salts are largely paramagnetic.

IV. MIXTURES OF SALT SOLUTIONS.

In view of the behavior of Heusler's alloys which exhibit a maximum permeability when the manganese and aluminium are present in the alloy in the ratio of their atomic weights, it seemed of interest to ascertain whether or not the susceptibility of a solution of manganese sulphate was affected by the addition of varying quantities of aluminium sulphate.

With this object in view a solution of manganese sulphate containing .0182 gram of manganese per c.c. was prepared, also one of aluminium sulphate containing .018 gram of aluminium per c.c., and one of copper sulphate containing .019 gram of copper per c.c., and the following mixtures of these solutions were examined.

(1) 8 c.c. $MnSO_4$ sol. + 8 c.c. H_2O.
(2) 8 c.c. $MnSO_4$ sol. + 2 c.c. $Al_2(SO_4)_3$ sol. + 6 c.c. H_2O.
(3) 8 c.c. $MnSO_4$ sol. + 4 c.c. $Al_2(SO_4)_3$ sol. + 4 c.c. H_2O.
(4) 8 c.c. $MnSO_4$ sol. + 8 c.c. $Al_2(SO_4)_3$ sol.

The susceptibilities of all the above solutions were found to be practically the same, showing that the aluminium present in the mixtures produced no modification in the susceptibility of the manganese such as it does in Heusler's alloys.

A second series was prepared in which the water added to make up the mixtures to 16 c.c. in the first series was replaced by the copper sulphate solution, and measurements made with these mixtures are given in Table VI.

TABLE VI.

Mixtures.	Susceptibilities $10^7 K$.	Atomic Ratio of Aluminium to Manganese.
I. $MnSO_4$ sol. 8 c.c. $Al_2(SO_4)_3$ sol. 8 c.c.	24.37	2.02
II. $MnSO_4$ sol. 8 c.c. Al_2SO_4 sol. 4 c.c. $CuSO_4$ sol. 4 c.c.	26.04	1.00
III. $MnSO_4$ sol. 8 c.c. Al_2SO_4 sol. 2 c.c. $CuSO_4$ sol. 6 c.c.	26.65	.50
IV. $MnSO_4$ sol. 8 c.c. $CuSO_4$ sol. 8 c.c.	27.49	——

A curve representing them is shown in Fig. 3, where the abscissæ represent the susceptibilities $\times 10^7$ and the ordinates the volumes of copper sulphate in the mixture.

These results show a regular increase in susceptibility proportional to the amount of copper added, which indicates that the susceptibilities of the mixtures followed a simple additive law, such as was found by Wiedemann[1] to hold for other salts. The susceptibility of aluminium sulphate has been shown to be extremely small, and on the basis of an additive law, its effect in the two sets of mixtures tested should be inappreciable. The susceptibility of any mixture should

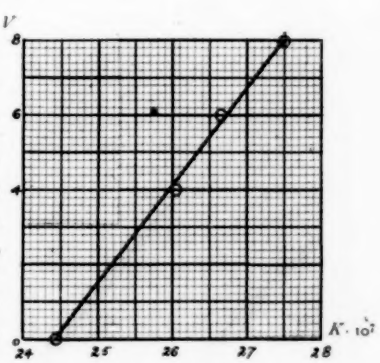

Fig. 3.

[1] Wied. Elect., III., pp. 955-972.

therefore be determined by the respective susceptibilities of the manganese and copper sulphates, and from the measurements given above it will be seen that this was the result obtained.

With the mixtures tested the atomic ratios of the aluminium to the manganese present in the solutions varied from 2.02 to zero. It is evident, therefore, from the above experiments that the relations the two metals bear to each other under the conditions of the present investigation are quite different from those which exist between them in the alloys. In the latter the aluminium by its presence produces a very pronounced increase in the permeability of the manganese but the measurements made in the experiments described above go to show that no such action takes place between the metals when solutions of their salts are mixed.

PHYSICAL LABORATORY,
UNIVERSITY OF TORONTO,
December 20, 1906.

RESEARCHES ON THE FORMS AND STABILITY OF AËROPLANES.

By W. R. Turnbull.

THE crying need to-day of aëronautics (considered as a science) is research work, and it is simply amazing, when one considers the thousands of scientific workers in all parts of the world devoting their time to refinements and ultra-refinements of well-known physical laws, to note how few persons are willing to take up the laws of aërodynamics and reduce them to an exact science.

The field that is open is alluring, even fascinating, and there is more scope for original research, and more chance for reward than in many of the beaten tracks; and yet the student of science still holds aloof, preferring to pass and repass over the footsteps of others.

Just as soon as the mechanical engineer can command at his designing table full data concerning the lift, drift and center of pressure of aëroplanes, the thrust and efficiency of different forms of air propellers, the strength and suitability of materials, and the complete elements entering into the longitudinal and transverse stability of an aërodyne,[1] considered as a whole, just so soon, and not before, can we reasonably expect the problem of aërial navigation to be really solved.

During a careful study of the history of aërodynes, which I have followed to the time of writing (October, 1906), the fact that has

[1] I use the word "aërodyne" in preference to "flying-machine," to denote an aëroplane-supported machine, driven by mechanical power through the air. I regret the introduction of a new term in the nomenclature of aërodynamics but there is certainly a term lacking to express the required meaning, for "flying-machine" (Ger. Flugmaschine: Fr. machine volante) is too general and too suggestive of a wing-flapping machine; "aëroplane" (Drachenschweber, Fr. aëroplane) should apply not to the whole machine but to the supporting surface only (moreover, aëroplane is used to denote a soaring machine, etc., which is not power-driven) and "aërodrome" (the term used by Langley) while it is euphonious really denotes an *air course* in the same sense that hippodrome means a course for horse-races. "Aërodrome" in France is also used to mean a balloon-shed (compare Capt. F. Ferber, Revue d'Artillerie, Mars, 1904, also a letter by the writer in The Scientific American, September 22, 1906).

most impressed me is that in every one of the proposed machines, both in the tried and untried ones, automatic longitudinal stability is not only a weak feature of the designs but is altogether absent in practically every case.

To a fairly satisfactory extent automatic transverse stability (by giving the aëroplanes a dihedral angle or by keels) is a solved problem, but not so automatic longitudinal stability. On this account I took up the laboratory study of aëroplanes with the aim clearly in view of finding if possible a type of aëroplane that should have *automatic* longitudinal stability. In this I have been most fortunate for I have found a type of double curvature aëroplane which not only possesses the elements of automatic longitudinal stability but also gives a very good "lift" and much reduced "drift" when compared with the single curvature surfaces most commonly employed and looked upon as our most efficient lifting aëroplanes.

To make the study systematic let us divide all the possible (*simple*) aëroplanes into five types as follows :

Type I. The plane surface.

Type II. The single curvature surface, convex on the under side.

Type III. The single curvature surface, concave on the under side (the opposite of type II.).

Type IV. The double curvature surface, concave at the forward portion, convex at the after portion, on the under side.

Type V. The double curvature surface, convex at the forward portion, concave at the after portion on the under side.

These are the only possible types (leaving out multiple curvatures of more than two) and we will take up the study of each individually and then compare the results.

In order to further simplify the problem and to make the results with the five different types strictly comparable with one another, I will deal only with surfaces having a rectangular plan-view and in which the width (at right angles to the air current) is twice the length (in the direction of the air current). While this method fails to cover the whole ground, since the number of possible plan-views is practically limitless, I feel assured that, *so far as general laws are concerned* the two-to-one rectangle is fairly typical of all forms.

Throughout the experiments, therefore, the five following planes

are alone used: See Fig. 1, in which the common (developed) plan-view is shown at *A* and the various cross-sections along any fore and aft line *ab* corresponding to the above mentioned types are shown at *B*, *C*, *D*, *E*, and *F*. The arrow gives the direction of the air current.

In each case the (developed) length is 8.485 inches and the width is 16.97 inches, giving a two-to-one ratio with an area of 143.99 square inches or practically one square foot.

These five types were experimented with, first as to their "lifts"

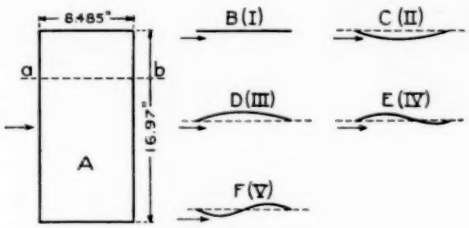

Fig. 1.

from 0° to 20°, second as to their "drifts" from 0° to 20° and third as to their longitudinal stability, or as to the position of the center of pressure at all angles from 0° to 90°.

Methods and Results.

General.

It is a well-known fact that the natural wind varies greatly (even from second to second) in intensity;[1] and it is even a difficult matter to obtain an artificial wind that has absolute constancy; in these experiments, therefore, in order that the results may be exactly comparable with one another, the writer employed a heavy 4-bladed fan-propeller to produce a constant air current and this fan-propeller was driven by an electric motor whose current was from a storage-battery of ample capacity. This fan-propeller was mounted at one (open) end of a rectangular box 6 feet long and 22 inches square and the speed of the propeller was so adjusted that the air-current at the opposite (open) end was exactly ten miles per hour at a distance of one foot from the end of the box. In the box near the propeller and for the first half of its length thin "baffle-

[1] Compare Langley, "The Internal Work of the Wind," and others.

plates" were placed to avoid rotary air currents so that a practically rectilinear air current was produced at the experimental end of the box. The box was further raised from the floor about two feet and this is a most important point in such experiments, for if the box be placed on or near the floor the intake of the fan-propeller is unequal with a consequent inequality at the opposite end of the box.

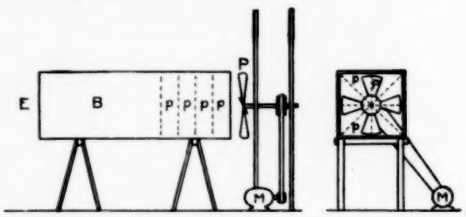

Fig. 2. Fig. 3.

Fig. 2 shows a side view and Fig. 3 an end view of the general arrangement of apparatus in all the experiments; in which *M*, is the electric motor; *P*, the propeller; *B*, the long box; *pp*, baffle-plates and *E*, the end at which the different apparatus for "lift," "drift" and center of pressure were placed.

The air current used in all the experiments was 10 miles per hour, for it was found (as before frequently established) that the "lift" and "drift" of the aëroplanes vary as the square of the speed of the air current, so that it is a simple matter to compute the "lift" and "drift" for any speed (within ordinary limits at least) when the values for a certain speed are known. The variation (if any) for the position of the center of pressure, with different velocity of air currents, was not determined, as the system was not sufficiently flexible to obtain high speeds.

The aëroplanes experimented with were made of thin tin plate with only a sufficient number of ribs (three) on the upper side, to make the forms absolutely rigid.

The original experiments were made in February to April, 1905, and were further repeated with refined apparatus in June and July, 1906.

"Lift" Experiments.

The apparatus for obtaining the "lift" (or vertical component of air pressure) of the aëroplanes, was mounted in such a way that

the centers of the aëroplanes were one foot from the free end of the box and exactly in the middle of the opening.

Fig. 4 shows a side elevation, and Fig. 5 an end elevation, of

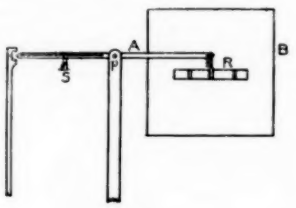

Fig. 4.

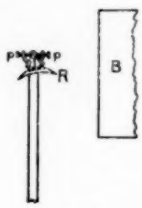

Fig. 5.

this apparatus which is simply a common balance arm, A (mounted on horizontal pivots, pp), with the aëroplane, R, mounted on one arm, and with a movable scale-pan, S, on the other arm to measure the loss of weight when the air current passes.

Throughout the experiments the aëroplane, R, was inclined at angles of 0°, 5°, 10°, 15° and 20° with the horizontal, and the corresponding settings of the scale-pan, S, to make a balance when the air current passes, were recorded.

In this way a series of readings were obtained which are plotted

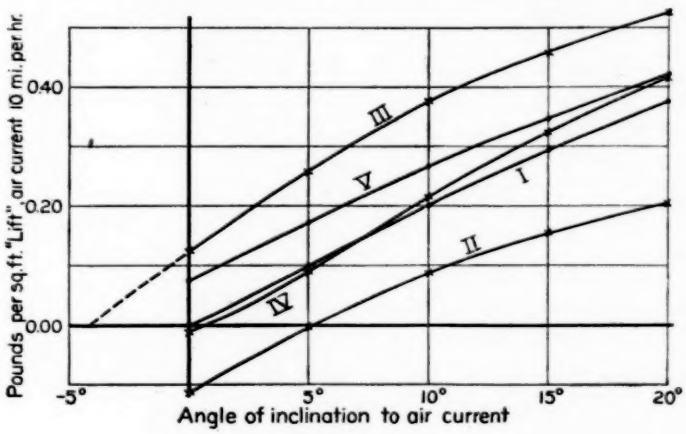

Fig. 6.

in Fig. 6, the average of several readings for each inclination of the aëroplane being used to determine several points on the curves.

It will be noted that the *comparative*[1] lift of Type V. at small angles is slightly greater than at large angles, by comparison with any of the other four types.

It will also be noted that the best "lift" (per se) is given by the single curvature surface, concave on the under side, and the poorest by the single curvature surface, convex on the under side; but, in selecting the best form of aëroplane, for use in an aërodyne, other factors enter as will be seen when these results are considered in connection with the "drift" and center of pressure experiments.

"Drift" Experiments.

The apparatus for measuring the "drift" (or horizontal component of the air-pressure) was mounted in such a way that the center of the aëroplane was one foot from the free end of the box and exactly in the middle of the opening.

Fig. 7 shows a side elevation and Fig. 8 an end elevation of this

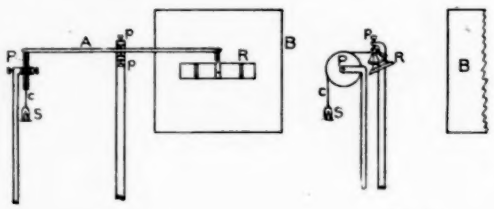

Fig. 7. Fig. 8.

apparatus, which is simply a balance arm, *A*, mounted on vertical pivots, *pp*, with the aëroplane, *R*, mounted on one arm and with the other arm attached to a cord, *C*, passing over a frictionless pulley, *P*. The scale-pan, *S*, in this case, being attached to the other end of the cord and the "drift" being measured by the adjustment of weights on the scale-pan.

As in the case of the "lift" experiments the aëroplane, *R*, was successively placed at 0°, 5°, 10°, 15° and 20° and the correspond-

[1] The writer has taken out patents in the United States, Canada, Great Britain, Germany and France on Types IV. and V., but by an oversight on the part of the patent attorneys the word "comparative" in referring to the lift of Type V. was omitted from a part of the description rendering the explanation of action absurd as there is no known form of aëroplane which will give a *greater* lift at a *smaller* angle, for ordinary angles between zero and thirty degrees.

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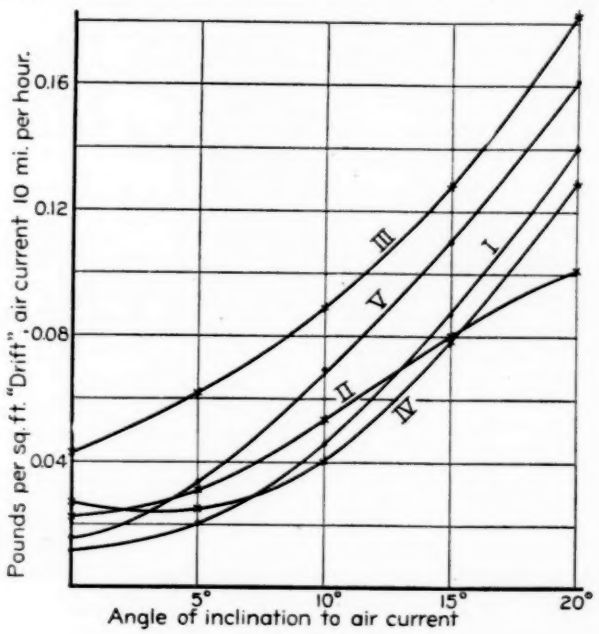
The data of the experiments on "drift" are plotted in Fig. 9, and on comparing them with the "lifts" of Fig. 6 we have some interesting results. It is noted that the single curvature surface (Type III.) concave on the under side, not only has the largest



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"lift," but also the largest "drift," while the "drift" for Type IV. (concave in front, convex in the rear, on the under side) actually decreases from 0° to 4°. For equal drifts we have the table given in the second following section and from this table we see that for equal "drifts" Type IV. at 10° has more than double the "lift" of Type III. at $-0.7°$. This fact has more than usual interest, be-

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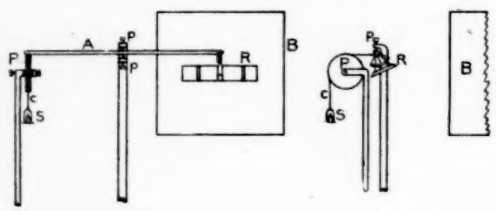

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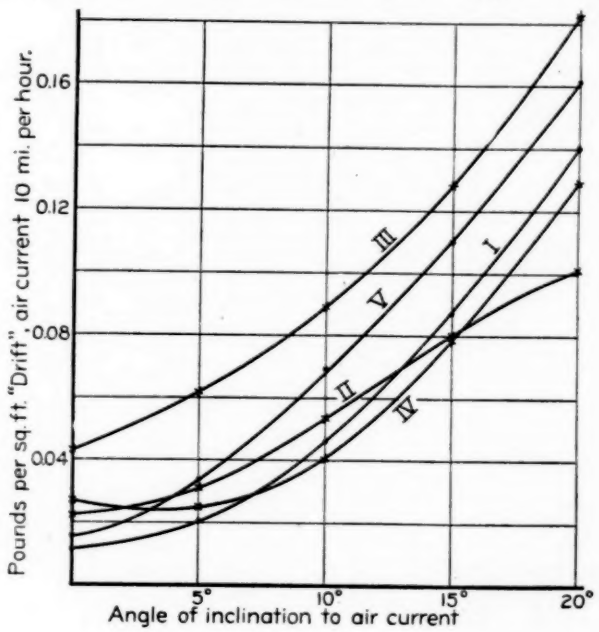

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cause it will be shown in the next section of this paper that Type IV. has automatic longitudinal stability from 90° to 0°, while Type III. has longitudinal instability between the angles of 28° and 0°.

The decreasing drift of Type IV. between the angles of 0° and 4° has also something to do with the soaring of birds, in the opinion of the writer, but this will be further referred to in another section.

In the two foregoing sections the writer has measured only the vertical component of air resistance ("lift") and the horizontal component of air resistance ("drift"). I have not followed Lilienthal's[1] method of plotting a resultant force (the total air resistance) from the "lift" and "drift" and afterwards resolving this resultant into a "normal" and "tangential" force; for these latter mainly apply in the case of an aëroplane falling through the air at the same time moving forwards along a course at a certain negative angle with the horizontal.

While Lilienthal's method is interesting when applied to his assumed slope of a bird's wing during the down-stroke, it is applicable only to artificial flight when it is sought to solve that problem by beating wings and it is, in general, not applicable to the problem of artificial flight with fixed aëroplanes driven horizontally through the air at a certain positive angle of inclination.

Center of Pressure.

Thorough in many ways as Lilienthal's results on "lift" and "drift" are, he unfortunately passed all too lightly over the more important question of the center of pressure, and while Joessel, Kummer and Langley experimented on this all essential problem, their experiments, invariably stopped short of those critical angles between 25° and 0° which are by all means the important ones in the study of natural and artificial flight.

The first method used by the writer, for measuring the center of pressure, was the usual one of the eccentric plane with counterpoising weight (used by Langley and others, see "Experiments in Aërodynamics," Langley, pp. 89–93, 1902) but its decided limitations were soon apparent and a delicate counterpoising spring was

[1] "Der Vogelflug als Grundlage der Fliegekunst," Berlin, 1889.

substituted with excellent results, for aëroplanes whose centers of pressure advance far in front of their centers of figure.

The counterpoising spring is limited however to use with certain aëroplanes between about 35° and 0° so that a use of both methods is necessitated for a complete set of readings between 0° and 90°.

The aëroplanes to be tested were ruled with lines (parallel to the leading edges) at distances of ½″, 1″, 1½″, etc., in front of (and behind) their centers of figure and at the points where these lines reached the side edges of the aëroplanes screw clamps were successively placed. The clamps being provided with pivot-holes for the insertion of the pivots of the supporting frame.

Figs. 10 and 11 show front and side elevations of the apparatus as employed with counterpoising spring; *M* being the arm holding the supporting frame, *F*. *F* carries small arms which are adjustable

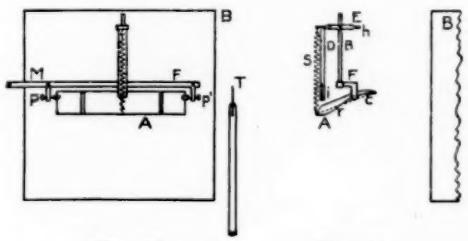

Fig. 10.

Fig. 11.

along its length to allow of use with different widths of aëroplanes, and these small arms carry the pivots, *pp'*, which carry the aëroplane under test, *A*. The frame, *F*, also carries a vertical rod, *B*, to which is clamped at any desired height, the block, *E*. *E* houses a horizontal rod, *H* (adjustable horizontally), which carries at its end a long sensitive spring, *S*, acting as a counterpoise for the unbalanced weight of *A*.

S is protected from the air-current by a shield, *D* (V-shaped in plain cross-section), and a small pointer soldered to *S* plays over the scale, *i*, near the bottom of *D*.

The dotted line, *C*, Fig. 11, shows the (imaginary) chord of the curvature of *A* and in every case the upper side of the rib, *r*, is made parallel to this chord so that the angle which *A* makes with the horizontal may be read by means of a vertically mounted trans-

parent protractor (shown in Fig. 10 at T) by sighting through the protractor along the upper line of the ribs.

When using the aëroplanes with a counterpoising weight the spring with its accessories was removed and the counterpoise secured by attaching a flat leaden weight to the upper side of the aëroplane under test.

Both methods of experiment were used as the circumstances of each setting of the pivots demanded and where the results of the two methods "overlapped" the accordance of the readings was satisfactory.

I have unfortunately been unable to obtain the original papers of Joessel and Kummer so that I do not know of their methods of experimenting but Langley mounted his counterpoised eccentric planes on the arm of a large whirling table and obtained (from a pencil trace) the angle corresponding to a certain setting of the side pivots and it is just here that the superiority of the method above described (with stationary aëroplanes and artificial air current) is most pronounced, for with several tpyes of aëroplanes (viz.: Types I., III., and V.) there is *not one angle but two* corresponding to one setting of the side pivots.

To illustrate this, let us take the example of plane No. III., with pivots on the center line; by Langley's method the only reading obtained is that at 90°; by the writer's method the reading for 90° is also obtained if the plane is untouched, but if the rear edge of the plane is supported loosely between the thumb and forefinger of one hand and carried to a position nearer the horizontal we find a second angle (at $12\frac{3}{4}°$) where, if the plane be slightly depressed, the rear edge will be strongly depressed, and if slightly raised, the rear edge will be strongly raised and we here have a second angle (capable of accurate measurement) *where the center of pressure again corresponds to the center of figure.* Of course, in place of the thumb and forefinger, a metal yoke with adjustable set screws could be similarly employed and could be vertically raised until this second critical angle was reached, but the thumb and forefinger furnish a sensitive index of the approach to this angle and the accuracy of the results are excellent.

The results of these experiments are given in Fig. 12 where

angles with the horizontal are plotted as abscissæ and centers of pressure (in percentage of the side of plane), both before and behind the center of figure, are ordinates.

In the case of the plane (Type I.) we note that the center of pressure steadily advances in front of the center of figure until the critical angle of 26° is reached when the center of pressure rapidly

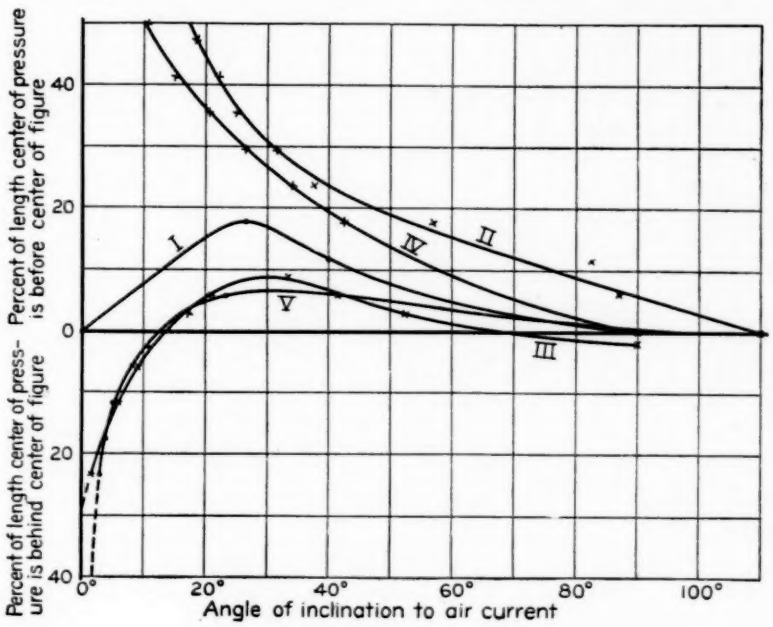

Fig. 12.

retreats to the center of figure as the angle becomes smaller and reaches 0°.[1]

In the cases of Types III. and V. this reversal takes place at somewhat near the same angle but at a much shorter distance in front of the center of figure, the center of pressure then rapidly retreats behind the center of figure which it passes at about 13°.[2]

[1] In the case of *a square* with 9″ side the writer found this critical angle at about 18° and it is just about this point where the curves of Joessel, Kummer, and Langley (who all experimented with square planes) cease. Joessel inferring that the law formulated by him, $c = 0.3(1 - \sin a)$ L held good until 0° was reached.

[2] My results for Type III. agree fairly well with those of Spratt who, according to Moedebeck's "Taschenbuch fur Flugtechniker und Luftschiffer," 1904, p. 335, has not made his experiments public. Spratts' results as given by Moedebeck are these: for a single concave curvature surface of $\frac{1}{12}$ curvature (shape not stated) the center of pressure

Type III., on account of its excellent "lifting" qualities, is the aëroplane that has been almost universally used in artificial flight experiments, but this reversal in the position of the center of pressure with its consequent utter lack of longitudinal stability is doubtless responsible for nearly all the accidents that have occured in aëroplane experiments, Lilienthal and Pilcher lost their lives through ignorance of this reversal and Langley's large machine was also probably wrecked through a lack of knowledge on this point.

For Types II. and IV. we have the remarkable result that the center of pressure steadily advances to the forward edge of the plane, where, by reference to Fig. 6, we note that the lifts are both negative. We therefore have, for these two types, *the condition of automatic longitudinal stability for all positive angles and "lifts."*[1]

SIGNIFICANCE OF THESE RESULTS FOR ARTIFICIAL FLIGHT.

In comparing the relative merits of these five types from the viewpoint of artificial flight, we must not only take into consideration their stability curves as shown in Fig. 12, but also compare the "lifts" and "drifts" as given in Figs. 6 and 9.

We then find that for the two aëroplanes of automatic stability Type IV. has a much superior "lift" to Type II. and a lesser "drift" between the angles of 2° and 15°; which are the important ones for artificial flight.

As pointed out above, Type III. has been almost exclusively used in artificial flight, but when it is compared to Type IV. we see that it has an utter lack of stability while Type IV. has automatic stability and while, at first sight, Type III. has the superior lifting qualities it also has a large "drift." As a matter of fact the "lift" of Type IV., as compared to its "drift," is superior to the "lift" of Type III., as compared to its "drift," *for certain angles* as per the table below, in which the respective values of "lift," at their corresponding angles, are given *for equal "drifts"* in each case.

at 0° lies near the rear side, it advances as the angle increases passing the middle point at about 17° and advances to about 8½ per cent. from the center of figure at 30°. It then moves in the opposite direction until it reaches the center of figure at 90°.

[1] Ahlborn, "Illus. Aeron. Mitteilungen, 1904. States *that only slightly convex* aëroplanes (Type II. of the writer's experiments) possess unconditional automatic stability, but his statement seems to be based on incomplete observations rather than on experiments.

Type IV.		Type III.		Lift IV.÷Lift III.
Angle.	Lift.	Angle.	Lift.	Ratio.
20°	.417	15.1°	.459	.91
19	.400	13.7	.439	.911
18	.381	12.4	.416	.916
17	.364	11.1	.395	.921
16	.346	9.6	.368	.941
15	.325	8	.331	.982
14	.303	6.5	.297	1.020
13	.283	4.6	.249	1.136
12	.261	2.9	.204	1.280
11	.240	1	.154	1.558
10	.216	−0.7	.105	2.055

From this table it appears that *for equal drifts*, Type IV. at 10° has actually double the lifting value of Type III. at −0.7°.

Lilienthal's principle for artificial flight was certainly well taken, viz.: "Stability first, propulsion afterwards" but we cannot review the history of experimental aëronautics without insisting on this revision: *Automatic stability first and always* propulsion so soon as prime-movers become sufficiently light.

Aëroplanes of Type IV. give the essential elements of automatic longitudinal stability; the well established principles of the "dihedral angle" and keels give automatic transverse stability; and gas engines, now approaching 3 pounds per horsepower give the required lightness of prime movers.

In these three principles, therefore, the problem of aërial navigation is theoretically solved. It now remains only to so combine these three essentials, in practical form, to solve the problem from an engineering standpoint.

I could hardly insist that aëroplanes of Type IV. are the only ones possessing the all important properties of automatic stability but among simple aëroplanes (*i. e.*, those having zero, single, or double curvature), my laboratory experiments have clearly shown that *Type IV. is the only aëroplane having at the same time automatic longitudinal stability combined with excellent lifting properties and exceedingly small drifting properties*, for all angles that would probably be utilized in artificial flight.

Significance of These Results to Natural Flight.

Lilienthal in his valuable book[1] has given an able analysis of bird flight, but he has made one assumption (page 76) which is hardly tenable, viz.: that the slightly concave surface is the form most like that of the bird's wing.

Neither my experiments nor a general consideration of the structure of the bird's wing lead to this conclusion.

The average cross-section of a birds wing when extended consists essentially of a forward portion, *A*, Fig. 13, of rigid formation, with a central portion, *B*, of partial rigidity, and a rear portion, *C*, of extreme flexibility.

While this formation certainly varies from the portion nearest the body to the extreme tip of the wings, I think it will be generally conceded that Fig. 13 fairly represents an average cross-sectiosn of an average bird's wing (during the down stroke).

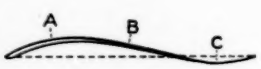

Fig. 13.

If now we conceive this wing in rapid up-and-down strokes passing through a resisting medium, such as the air, it requires only a rudimental conception of mechanics to infer that in the up stroke the cross-section assumes the form shown in *AC*, Fig. 14, while in the down stroke a form similar to that shown in *AC*, Fig. 15, would be necessitated.

In each figure, *op*, gives the direction of the wing motion in each

Fig. 14.

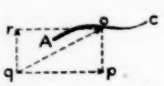

Fig. 15.

case, while *or* represents the absolute direction of the air current. It is obvious that the *relative* movement of the air particles is roughly represented by *qo* and it is equally obvious that a wing, of the type described in connection with Fig. 12, would be forced into the approximate forms shown in Figs. 14 and 15 — *corresponding to the two Types II. and IV. both of which have been shown by the laboratory experiments to have automatic longitudinal stability.*

If then we accept this reasoning, it is proven that in bird flight

[1] "Der Vogelflug als Grundlage der Fliegekunst," Berlin, 1889.

the two forms assumed by the wing *both have automatic longitudinal stability*, that in the up stroke the "lift" (negative in this case) is a minimum (see Type II., Fig. 6) while the "drift" is small (see Type II., Fig. 9) and that in the down stroke the "lift" is excellent (see Type IV., Fig. 6) while the "drift" is a minimum (see Type IV., Fig. 9).

From this it is seen that the laboratory experiments are in full accord and give a most rational explanation of the wonderful properties displayed in the flight of a bird.

For soaring or sailing flight the most probable form of the bird's wing is that shown in Fig. 16 which is simply a less accentuated form of Fig. 15, due to the lessened air pressure on the under side of the wing.

The resolution of forces as shown in this figure gives a perfectly rational explanation of soaring flight, by which a bird is supported and at the same time advances against an ascending current of air, for in working out the "lift" and "drift" under these conditions we find that OR, the resultant, is in advance of the normal to the chord of the double curvature,

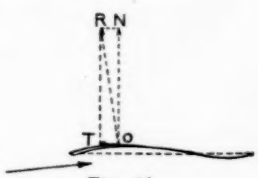

Fig. 16.

and we have for the forces acting upon the surface the vertical component ON equal to the weight of the bird and the tangential force, OT, which furthers, instead of hinders, the advance against the air current.

It has been frequently shown by delicate anemometer measurements, etc., that no natural wind is steady but that the natural air currents have a pulsating movement, with pulsations varying sometimes several times in a second. The soaring bird can instinctively feel these pulsations and can make excellent use of them by slightly inclining the extended wings more or less to the pulsating current and can thus maintain or increase its height while at the same time it slowly advances against the air current.

Experiments from many sources (Lilienthal and others) have shown that there is also in natural winds, near the surface of land or water, an ascensional trend of about 3° and this further facilitates the soaring of most birds.

Referring again to the automatic stability of birds, the extended tail in many species is undoubtedly a valuable *auxiliary* to the maintenance of automatic longitudinal stability; but it is quite evident from a consideration of several species in which the tail is almost lacking (*e. g.*, the wild duck) that the tail is not essential, and this further accentuates the idea that the primal and main cause of automatic stability in bird flight is to be found in the wing-forms shown in Figs. 14 and 15.

Lilienthal's close adherence to his conception of the natural wing form was the probable cause of his death, for in all of his soaring machines (and in a great majority of soaring machines that have since been built) the single concave surface has alone been used and with it of necessity (see Curve III., Fig. 12), automatic stability is impossible and "stability" is only obtainable by the agility of the operator in moving his body or an auxiliary steering aëroplane.

THE INFLUENCE OF AN UNDERPLANE.

In considering the subject of aëroplanes used in artificial flight it occurred to the writer that the lifting properties of aëroplanes might

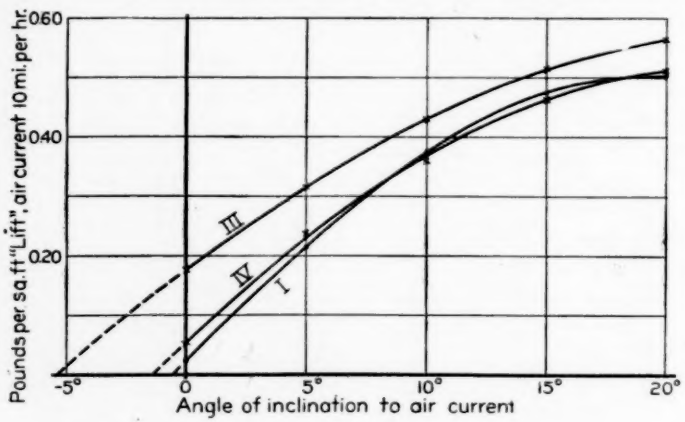

Fig. 17.

be considerably increased by allowing the aëroplanes to skim close to an underplane such as a level piece of land or close over water.

As is shown in Fig. 17, a considerable increase in "lift" really occurs (due no doubt to the wedging action and consequent compression between the aëroplane and underplane) and at first sight

it would seem a valuable consideration in the design of an aërodyne; since it is just when arising and alighting, when the speed would be comparatively low, that the maximum "lift" would be desired. However, many other modifying conditions must be considered, since screw propellers must have a considerable diameter and their central line of thrust must be in line with the aëroplane resistance; and, moreover, the "drift" is also considerably increased with an underplane and the center of pressure is shifted, in general, considerably farther "aft" than it would be in free flight, introducing serious difficulties in balancing. These various drawbacks of the underplane may be overcome (*e. g.*, by superimposed planes), but it is well to draw attention to them.

In Fig. 17 the "lift" curves, with an underplane, are given for Types I., III., and IV. The readings with the underplane were obtained with the apparatus shown in Figs. 4 and 5 under the same conditions as before, but with an underplane (a large drawing board) placed horizontally one half inch below the rear edge of the aëroplane in its mean position of balance.

It is needless here to go into details concerning the influence of an underplane on the "drift" and center of pressure of aëroplanes, but the writer's experiments with "lifts" may be summarized as follows: (1) With plane surfaces (Type I.) and double curvature surfaces (Types IV. and V.) the *relative increases* of "lifts" with an underplane are greater than in the case of single curvature surfaces. (2) Under certain conditions the lift may be nearly doubled by an underplane very close to the rear edge, but in general, the influence nearly disappears at a distance above the underplane equal to about one third the length (in the direction of the air current) of the aëroplane.

In conclusion, with regard to the general design of a practical aërodyne, I can only say that the proportion of parts is an essential feature which must be worked out by outdoor experiments, preferably over a sheet of water.

The laboratory experiments, described in this paper, can give but little help in the proportioning of parts, but they clearly show the type of aëroplane to be adopted and the type to be distinctly avoided. On the one hand we have Type IV., with its great advantages of

automatic longitudinal stability, excellent lifting properties and exceptionally low "drift" and on the other hand the long-adopted aëroplane with single concave curvature, which, while it has most superior lifting qualities, has also a most serious "drift" (for high speeds particularly to be considered) and *above all in importance, most treacherous longitudinal stability.*

A trifle in added speed gives the former type equal "lift" with the latter (since the lift and drift increase with the square of the speed), so that it should be an easy matter for a designer to choose unhesitatingly between the two.

ROTHESAY, N. B., CANADA,
October 10, 1906.

ON THE NATURE OF OPTICAL IMAGES.

By Albert B. Porter.

THERE is no doubt that the teaching of mathematics has suffered from the somewhat arbitrary division of the subject matter into what have been aptly termed "water-tight compartments." To a lesser degree the same thing is true of physics, if current teaching may be judged by some of the current texts. Thus in the treatment of light, for example, although the sharp distinction between geometrical and physical optics has largely passed with the passing of the old-fashioned (but often indispensable) ray-method of treating the formation of images, yet the wave-method which has replaced it is sometimes presented in a form almost as purely geometrical. We have abandoned the diagrams in which the so-called rays were frankly intended to represent nothing more than directions of propagation, and have substituted figures showing an impossible array of sharply limited wave-fronts primly marching toward a focus, with never a trace of diffractive fraying at the edges. Even in the few text-books with which I am acquainted in which this fraying is taken into account in an investigation of the effects of aperture on resolving power, the discussion follows so far after the study of image formation that the student must almost inevitably absorb the idea that diffraction and interference merely act to modify an image otherwise formed. Nor do I anywhere recall in the elementary books an explicit statement to the effect that all optical images arise by interference and are indeed particular cases of interference patterns. That this is, however, true appears at once from a consideration of the method of propagation of light by means of secondary wavelets, which spread in all directions from every point of the wave-front in such a manner that, were it not for interference, each point in the image would receive light from all points in the object. Dark places in the image are thus dark because of destructive interference, and light parts are bright on

account of additive interference. All this, though of course sufficiently familiar to the teacher, may be far from evident to the student, and this is my excuse for describing a simple and quickly arranged experiment which shows the relation between interference and image formation in a manner so striking as not to be easily forgotten.

The experiment consists in passing a parallel beam of monochromatic light through a coarsely ruled, black-line, diffraction grating, and then through a convex lens. On the far side of the lens a system of sharply defined interference fringes is formed which can be seen by aid of an eyepiece, or intercepted on a screen, at any point over a considerable range along the axis. Somewhere in this system of fringes is the geometrical image of the grating, but it is visually quite indistinguishable from any other transverse section of the fringe system. Clearly in this case, the geometrical image is merely that section in which the geometrical condition of similarity to the object is satisfied.

The best arrangement of the experiment is the following. Light from an arc lamp *A* is focused by means of the lens *B* upon a narrow slit *C*. Thence it passes through a direct-vision prism at *D*,

Fig. 1.

and the spectrum is focused by the lens *E* upon the narrow slit of a collimator *FG*. The parallel beam of monochromatic light thus obtained falls upon the mirror *H* of a microscope *KJ*, upon whose stage, at *I*, the grating is placed in such a position that its ruled lines are parallel to the projections of the two slits *C* and *F*. Using a black-line grating of 400 lines to the inch, and having both slits narrowed down to a small fraction of a millimeter so as to secure very homogeneous illumination, the field of view was examined

with ⅔ inch objective and 1 inch eyepiece. The interference fringes appeared in the field of the eyepiece with exquisitely sharp definition throughout the whole range of the coarse adjustment of the microscope, *i. e.*, over a distance of 58 mm., beginning with the front of the objective in contact with the grating and with its focal plane 7 mm. below the ruled surface; and the fringes could be traced through a much greater range by withdrawing the eyepiece and moving it back along the axis. As the microscope is slowly focused upward, the bands undergo curious changes in appearance, the lines showing sometimes close together and again farther apart, but the definition is almost equally sharp throughout the whole range of adjustment, so that any section of the fringe system is as good an apparent image as any other section. Similar but less perfect effects may be obtained by illuminating the field by means of sodium light passing through a slit a couple of millimeters wide at a distance of one or two meters.

If the angle of the incidence of the light on the grating is changed by moving the mirror, the whole fringe system shifts to one side or the other except in the focal plane, where it remains stationary. This shows (1) that the focal plane is the plane in which the interference fringes formed by light of all incidences coincide; (2) that, when a broad source is used, the geometrical image is really a superposition of coincident interference patterns; and (3) that the usual absence of a sharp image outside the focal plane is due to the more or less uniform illumination resulting from the overlapping of fringe systems due to light coming from various points in the source. When the grating is illuminated by a parallel beam of white light by means of a collimator with very narrow slit or, less perfectly, by a distant gas flame turned edgewise, the effects are similar except that outside the focal plane the fringes are colored. Hence (4) the focal plane is also the plane of achromatic interference, *i. e.*, the plane in which the fringes due to light of various wave-lengths coincide.

These experiments show very clearly why it is in general essential to use a condenser to illuminate the field of a microscope in order to obtain a critical image, *i. e.*, an image which comes sharply into and out of focus and which is hence as free as possible from confusion with details of structure lying above and below the focal

plane. It is interesting to observe how, as the illumination is made less homogeneous and more convergent, the distance through which the interference bands can be distinguished decreases, and the precision of focusing increases.

In the case of self-luminous objects, although the geometrical image is still to be considered as an interference pattern, the effects outside of the focal plane are greatly modified by the absence of definite phase relations between the waves emanating from various points.

RADIATION FROM SELECTIVELY REFLECTING BODIES.

By W. W. Coblentz.

IN his memoir on infra-red reflection spectra the author[1] shows that all the silicates examined have bands of strong selective reflection in the region of 8 to 10 μ and discusses the effect this would have upon a reflecting surface like the earth or the moon which is largely composed of silicates. Since this discussion has brought forth criticism, favorable and unfavorable, in print and in private communications, the purpose of the present paper is to summarize, in a general way, the present data bearing upon the subject, and to discuss certain points not mentioned previously.

The Effective Temperature of the Sun.

It has been shown by Poynting[2] that, when a surface is a complete radiator and absorber, its temperature can be determined at once by the fourth power law, if we know the rate at which it is radiating energy. If it is radiating what it receives from the sun, then a knowledge of the solar constant enables us to find the temperature, which will be the highest the surface can attain when it is receiving heat only from the sun. Knowing the solar constant and the radiation constant ($\sigma = 5.32 \times 10^{-5}$ ergs Kurlbaum[3]) Poynting computes the effective temperature of the sun. If s is the radius of the sun's surface, and R the radiation per square centimeter then the total rate of emission is $4\pi s^2 R$. This must equal the radiation passing through the sphere of radius r, at the distance of the earth, and with a surface $4\pi r^2$ gives

$$4\pi s^2 R = 4\pi r^2 S$$

where S is the solar constant.

[1] Investigations of Infra-Red Spectra, Parts III. and IV. Published by the Carnegie Institution of Washington, December, 1906.

[2] Phil. Trans. Roy. Soc. London, Vol. 202 A, p. 525, 1903. Of course no surface is a complete radiator or absorber of energy.

[3] Kurlbaum, Wied. Ann., 65, p. 748, 1898.

Hence, $R = 46{,}000\,S$. Using the solar constant $S = 2.5$ calories $= 0.175 \times 10^7$ ergs per sq. cm. per sec., $R = 0.805 \times 10^{11}$.

If we equate the sun's radiation to $\sigma\theta^4$ where σ is the radiation constant, we get θ, the "effective temperature" of the sun, that is, the temperature of a full radiator which is emitting energy at the same rate.

Thus

$$5.35 \times 10^{-5}\theta^4 = 0.805 \times 10^{11}$$

whence $$\theta = 6{,}200^\circ \text{ Abs.}$$

With each new determination, however, the solar constant is being reduced from the former high values, so that the more probable value [1] is about 2.1 gr. cal./cm.^{2} min.

Using this value of $S = 2.1 = 0.147 \times 10^7$ ergs/cm.^{2} sec., $R = 0.676 \times 10^{11}$.

Thus

$$5.32 \times 10^{-5}\theta^4 = 0.676 \times 10^{11}$$

whence $$\theta = 5{,}980^\circ \text{ Abs.}$$

This is somewhat closer to Wilson's [2] value, 5,773° Abs. which he obtained by making a direct comparison of the radiation from the sun with that from a "full radiator" [3] at a known temperature.

The Limiting Temperature of the Surface of the Moon.

Poynting (loc. cit.) further shows that when there is no conduction inwards from the surface, the highest temperature of a full radiator is attained when its radiation is equal to the energy received. Equating the energy to the solar constant using

$$S = 0.175 \times 10^7$$

$$5.35 \times 10^{-5}\theta^4 = 0.175 \times 10^7$$

whence $$\theta = 426^\circ \text{ Abs.}$$ (Moon's temperature).

[1] Abbot, Smithsonian Miscellaneous Collections, Vol. 45, p. 81, 1903.

[2] Wilson, Proc. Roy. Soc., Vol. 69, p. 312, 1901. Fery and Millochau (C. R., 143, p. 570, 731, 1906) using a pyrometer found the temperature of the solar surface to vary from 5888° to 5963° Abs. with a mean value of 5620° Abs.

[3] Poynting (loc. cit.) very aptly says: "A surface which absorbs and therefore emits every kind of radiation, is usually described as 'black,' a description which is obviously bad when the surface is luminous. It is much better described as 'a full absorber' or 'a full radiator,'" *i. e.*, a complete radiator, as distinguished from a partial radiator or so-called "non-black body." In justice to Kirchhoff who was the first to give a clear discussion of this subject it should be called the *Kirchhoff Radiator*.

If part of the energy is reflected and only a fraction x of that falling on it is absorbed then the effective temperature is $x^{1/4} \times 426$ Abs. From Langley's[1] estimate that the moon absorbs $x = \frac{7}{8}$ the energy it receives, Poynting (loc. cit.) computes upper limit of temperature of the surface exposed to a zenith sun to be

$$\theta = 426 \times (\tfrac{7}{8})^{1/4} 412° \text{ Abs.}$$

But this upper limit to the temperature of the hottest part of an airless planet is never attained because the moon turns the same face to the earth instead of to the sun. He shows that, if N is the normal stream of radiation from a unit of surface of the moon immediately under the sun, the normal stream from the equivalent flat disk is $N_d = 2/3\ N$. "The effective temperature of the flat disk is therefore $\sqrt[4]{2/3}$ that of the surface immediately under the sun at the same distance from it. Then the effective average temperature $= 412 \times \sqrt[4]{2/3} = 371°$ Abs. The upper limit then, to the average effective temperature of the moon's disk is just below that of boilnig water." If we use $S = 0.147 \times 10^7$ instead of 0.175×10^7.

$$5.32 \times 10^{-5}\theta^4 = 0.147 \times 10^7$$

whence $\theta = 408°$ Abs. Hence $x^{1/4} \times 408 = 395°$ Abs. and the "effective average temperature" is $395 \times \sqrt[4]{2/3} = 355°$ Abs. or 82° C. for the full moon. This assumes no conduction inwards. Evidently there is an appreciable amount of heat conducted inwards, for the results of Langley (loc. cit.) and of Very[2] show that the surface of the full moon is about 300° Abs.

Fall of Temperature of the Moon During Eclipse.

Langley (loc. cit.) made observations on the eclipse of Sept. 23, 1885, which indicate a sudden and very rapid fall of energy from the moon as the eclipse commenced, with some indications of a rise nearly as rapid after its conclusion. In Fig. 1 are plotted his observed galvanometer deflections during the progress of the eclipse. The observations were interrupted by the formation of clouds just at the

[1] Langley, Nat. Acad. Sci., Vol. 4, part 2, p. 197.

[2] Very, Astrophys. Jour., 8, p. 199, 1898. Very, however, shows that the maximum temperature may be 455° Abs. for limited regions of the moon's surface, and (p. 286) a a mean temperature of about +97° C.

predicted time for the moon to leave the umbra. The curve shows that in the short time of about 1.5 hours, in passing from the penumbra to the umbra, the radiation from the west limb has fallen from a maximum to a zero value. In other words the fall of temperature is practically coincident with the change in illumination, and at first appeared to the writer[1] to indicate that the greater part is reflected energy. That the moon at mid-eclipse is still as warm as the earth is shown by the fact that the galvanometer gave zero deflections. If the moon had been cooler than the earth then the deflections would have been negative. Subsequent search of the literature on the subject shows that Very[2] found appreciable radiation from the moon during totality. The following computations show that this is to be expected. After about 11 days of insolation the temperature of the sun-lit surface of the moon will be fairly constant. From the surface inwards there will be a layer which will be at a uniform temperature, for the period of 1.5 hours, as compared with 11 days. If then the moon were suddenly eclipsed find the fall of temperature of the surface with time ($x = 0$, $\theta = f(t)$, assuming at time $t = 0$, $\theta_0 = 300°$ Abs.) is found from the equation:

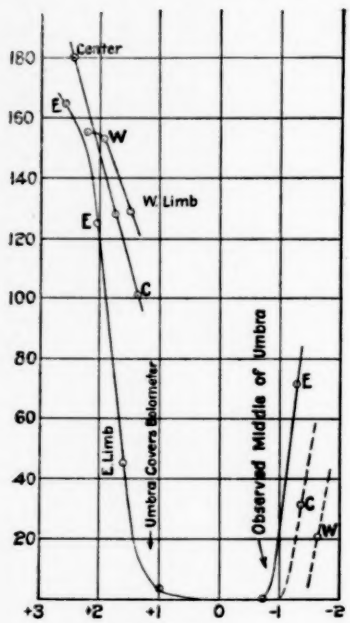

Fig. 1.

$$k\frac{d^2\theta}{dx^2} - \sigma\theta^4 = \rho s\frac{d\theta}{dt}$$

where $k =$ conductivity, $\rho =$ density and $s =$ specific heat. The dependent variable enters here as a fourth power, and makes the solution difficult. The computations are made for instantaneous eclipse, while in the actual case the shadow moves slowly across the

[1] PHYSICAL REVIEW, 23, p. 247, 1906.

[2] Very, Prize Essay on the Distribution of the Moon's Heat, p. 40.

surface. Hence the error due to neglecting conductivity is partly compensated by the slow eclipse.

By neglecting conductivity from the interior, the temperature of the surface will fall more rapidly and we have a steeper temperature decadence curve. The solution of the equation is very simple if we neglect conduction and is

$$\theta = \frac{\theta_0}{\sqrt[3]{\frac{3\theta_0^3 \sigma t}{\rho s} + 1}}$$

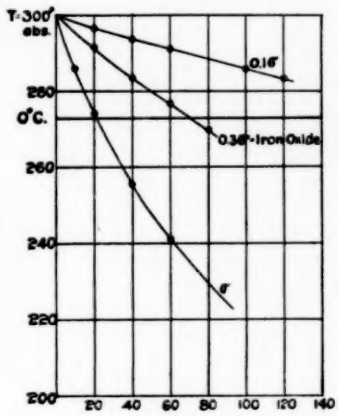

Fig. 2.

For t in minutes $\sigma = 76.6 \times 10^{-12}$ gr. cal./cm.^{2} min. $\rho = 2$, and $s = 0.2$ (approximate values for rock material). Hence

$$\theta = \frac{300}{\sqrt[3]{1 + 0.0155t}}$$

for a complete radiator.

In Fig. 2 are plotted the temperature decadence curves for the radiation constant σ, for $0.3\,\sigma$ (emissivity of iron oxide) and for $0.1\,\sigma$. The curves show that for a complete radiator the temperature would fall to 280° (room temperature when the bolometer would be in temperature equilibrium with the moon) in 17 minutes, for $0.3\,\sigma$ in 55 minutes, and for $0.1\,\sigma$ in 140 minutes.[1] By including the conductivity term these periods would be considerably prolonged. The present solution is close enough, however, to show that the temperature decadence curve is not coincident with the eclipse curve, unless the emissivity is of the order $0.3\,\sigma$. Hence, the writer's[2] previous surmise that at the beginning of totality radiation should still be appreciable is substantiated, although the observations made by Langley, Fig. 1, seemed to contradict it. Very's observations[3] show that during totality of the eclipse of January 17, 1889, the radiation from the umbra of the eclipsed moon was about 1 per cent. of the heat which was to be expected from the full moon.

A computation was also made assuming the temperature to be

[1] The solution for $0.1\,\sigma$ and $0.3\,\sigma$ is, of course, only approximate since the emissivity varies more nearly as the 5th power instead of the 4th power, as here used: and the computed temperatures should be somewhat higher.

[2] PHYS. REV., 23, p. 247. 1906. [3] Very, Distribution of the Moon's Heat, p. 40.

350° Abs. Here for σ, the temperature would fall to 288° in 38 minutes and for 0.3σ in about 130 minutes.

SELECTIVE REFLECTION OF SILICATES COMPARED WITH REFLECTION FROM THE MOON.

The writer (loc. cit.) has examined the infra-red reflecting power of a series of materials including quartz, feldspars, amphiboles and micas, which are the chief constituents of the rocks of the earth's crust and has found a uniformly decreasing reflection power of 3 to

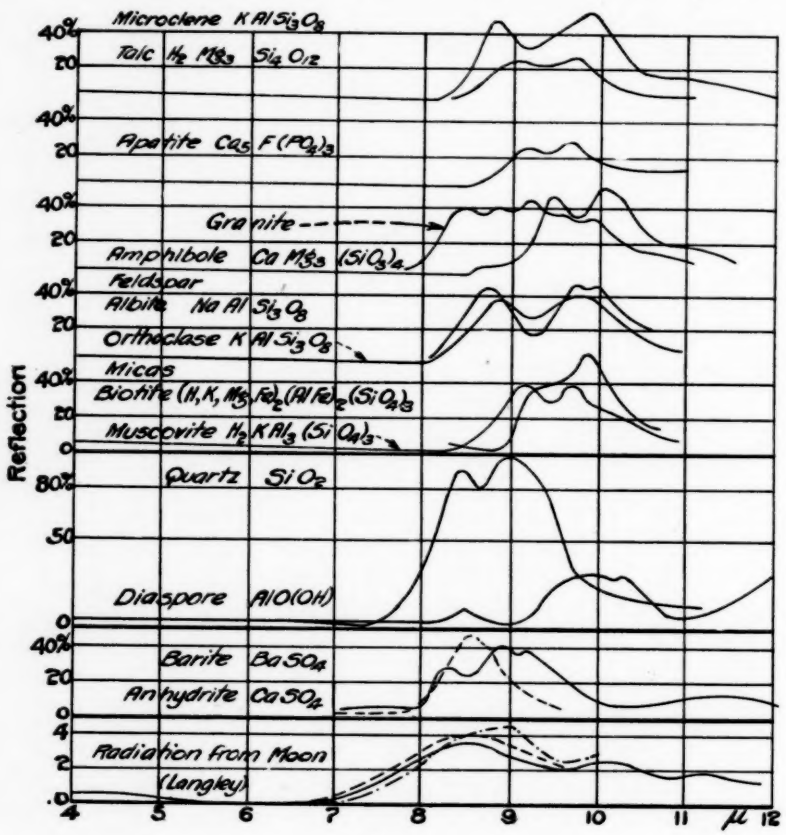

Fig. 3.

5 per cent. at 4 μ to 0.3 per cent. at 7 μ, followed by bands of metallic reflection from 8.5 to 10 μ. In this latter region the mean reflecting power is from 40 to 50 per cent. for silicates while pure quartz has a reflecting power of 90 per cent. The reflection curves are given in Fig. 3, in a condensed form. In the lower part of the

figure several of Langley's lunar radiation (reflection) curves are given, in which the ordinates are in arbitrary units. Of course, the absolute reflecting power of the moon is unknown. Zöllner has shown that, owing to the irregularities of its surface, the full moon does not reflect as a smooth sphere would do, but very nearly as a flat disk of like reflecting power and filling the same angle. Such a disk, if it diffused all the solar energy which falls on it, would send to us 1/97,300 of what the sun does provided it reflected perfectly, in all directions, the total incident solar energy. The moon, however, is not such a reflector, and in the visible spectrum the solar light is about 500,000 times moonlight. On this scale the curves of the moon given in Fig. 3 would not be visible. The depression in the lunar curves at 9.6 μ, is due to an atmospheric water band. The region from 5 to 7 μ likewise contains water bands. One can readily see from these curves that a reflecting surface composed of rocks such as granite, basalt, diorite, etc., which are mixtures of quartz, feldspar, hornblende and mica would give a curve similar to that of the moon, *i. e.*, that part transmitted by the earth's atmosphere between 8 and 12 μ.

The general assumption is that the crust of the earth and of the moon have the same constitution, viz.: a series of silicates. It follows from the present discussion that the earth and the moon may be considered selectively reflecting surfaces, with a band of metallic reflection from 8.5 to 10 μ. Hence, in the region of the spectrum from 4 to 7 μ only 3 to 5 per cent. of the radiation from the sun would be reflected from the moon, under the best surface conditions (diffuse reflection), while in the region from 8 to 10 μ the reflecting power would rise to 50 or even 80 per cent. Under actual conditions the balance will still be in favor of the region of 8 to 10 μ. In the region of 7 μ the reflecting power falls below 1 per cent.

Reflection and Radiation from the Moon.

From an examination of some of the discussions of the reflection of the moon it appears that a matte surface cannot be selectively reflecting. In fact the whole misunderstanding seems to hinge on this point. Diffuse reflection follows from directed (or scattered) radiation falling upon small reflecting surfaces, the planes of which lie in all directions. For each infinitesimal surface, the ordinary

laws of reflection are obeyed in full, unless the linear dimensions of the reflecting surface, or of the rugosities or inequalities on it, are small compared with the wave-length. In diffuse reflection (see Wood's Optics, p. 36 for examples) the irregularity decreases as the angle of incidence increases, so that for a given roughness we get regular reflection.[1] The long waves will be reflected first, then the shorter. But a surface consisting of a substance like quartz, which reflects like a metal for $\lambda = 8.5$ and $9.03\ \mu$, and like a transparent medium for all other wave-lengths, must still be selective for diffuse reflection. We are not so much concerned with the question as to where the maximum emission of the moon occurs as we are in the fact that, in the region of 8 to $10\ \mu$, where Langley observed radiation from the moon, there is also reflected energy from the sun.

Assuming that the temperature of the moon is 350° Abs. and the temperature of the sun[2] is 5,900° Abs., the problem is reduced to the calculation of the relative intensities of the energy emitted and reflected by the moon at 8 to $10\ \mu$. From the transmission curves of the water vapor (Fig. 6) it will be seen that in the region from 5 to $8\ \mu$ almost all the energy will be absorbed by the earth's atmosphere. The reflecting power of the moon for visible rays, according to Langley (loc. cit.), is only 1/500,000 full sunlight. Assuming that at $9\ \mu$ the reflecting power is, on an average, 10 times that at 0.5 to $4\ \mu$ (a low estimate), the value becomes 1/50,000 or 0.00002 per cent.

Using the above values for the temperature of the moon and of the sun, from Planck's[3] formula for the distribution of the energy

[1] Apropos of this discussion, Very (Ap. Jour., 8, p. 278, 1898) records that the percentage of reflected rays, as measured by a bolometer, is "especially large (perhaps two or three times the usual proportion) in the narrow crescent moon where the rays suffer a grazing reflection at a large angle of incidence, the emission under the corresponding angle of emission being small."

[2] This value is about the mean of the observed and computed temperatures. Abbot's (loc. cit.) solar spectrum energy curves deduced from bolometric measurements give us further data for computing the temperature of the sun. The published curve is drawn very asymmetrical and is much depressed on the side of the short wave-lengths. The data given permit one to draw a more symmetrical curve (as one would expect it to be) which shifts the maximum wave-length from 0.49μ to 0.46μ. From $\lambda_{max} T = 2920$ the latter value ($\lambda = 0.46\mu$) gives a "black body" temperature of about 6300° Abs. If the sun's radiation is purely thermal it must lie between that of a complete radiator and that of some highly refractory substance like bright platinum, for which the constant is 2630 instead of 2920. Using 2630 the computed temperature is $T = 5700°$ Abs.

[3] Planck, Verh. d. Deutsch. Phys. Ges., 2, p. 202, 1900.

in the spectrum of a full radiator (which, of course, the moon is not)

$$E = c_1\lambda^{-5}(\varepsilon^{c_2/\lambda T} - 1)^{-1}$$

we can obtain the ratio of the intensities for the two temperatures $T_1 = 350°$ Abs. and $T_2 = 5{,}900°$ Abs. from the formula

$$(\varepsilon^{c_2 \div \lambda T_2} - 1) \div (\varepsilon^{c_2 \div \lambda T_1} - 1)$$

where $c_2 = 14{,}500$ and λ max. $= 9\,\mu$ the ratio is 0.00316. But the moon, not being a perfect radiator, will have a smaller emissivity at 8 to $10\,\mu$. If its surface were iron oxide [1] its emissivity would be only 0.3 that of a Kirchhoff radiator, and for the region at $9\,\mu$, judging from the drop in the emission curve (Fig. 5) the emissive power may be less than this, say 0.1. This ratio of the emissive power of the moon to that of the sun will then be 0.000316, which is 16 times (0.000316 ÷ 0.00002) the reflected energy of the sun from the moon. Computations like these, however, which require all sorts of assumptions, can be of little value [2] ultimately. As matters now stand, we know that Langley observed also direct radiation from the sun, in this region of the spectrum, and from existing data of the radiation from the moon we do not know how much of it is selectively reflected energy from the sun. In the visible spectrum the observed energy will consist of that reflected from the surface and that part which enters the particles and is totally reflected. In the region of 8 to $10\,\mu$ (for silicates) almost all of the observed energy will be reflected from the surface (no internal reflection), hence the above computations can be only rough approximations.

Emission from a Partial Radiator.

We have now to consider a problem which has heretofore not been discussed in connection with the radiation from the moon. Planck's formula for the intensity of radiation at a given wave length, λ, in the spectrum of a full radiator is

$$E_\lambda = c_1\lambda^{-5}(\varepsilon^{c_2/\lambda T} - 1)^{-1}.$$

For any radiating body the emission

$$\varphi_\lambda = A_\lambda E_\lambda$$

[1] Kayser, Spectroscopy, Vol. 2, p. 80.

[2] Very, Astrophys. Jour., 24, p. 353, using different data finds a larger ratio than above.

where A is the absorption coefficient. But $A_\lambda = 1 - R_\lambda$(for non-conductors) where R is the reflecting power, and hence, $\varphi_\lambda E_\lambda (1 - R_\lambda)$.

It follows therefore that a selectively reflecting body like quartz will be a partial radiator and that if we compare its radiation curves with that of a full radiator, there will be *minima* of *emission* in the region of 8.5 μ and 9.03 μ. This was predicted by Aschkinass[1] and experimentally verified by Rosenthal,[2] with quartz mica and glass. The latter found the observed emission curves of these substances to coincide precisely with the computed curves, using the observed reflecting power of these minerals and the observed emission curve of a complete radiator at the same temperature.

Since we have constantly before us examples of selective emission, such as the Welsbach mantle, and high temperature radiation of such metals as tungsten and tantalum it is of interest to consider the radiation of several substances having bands of selective reflection.

In Fig. 4 is given an illustration of the marked effect of selective reflection upon emission. The extraordinary reflection of carborundum, SiC, curve *a* has been found in only one other sub-

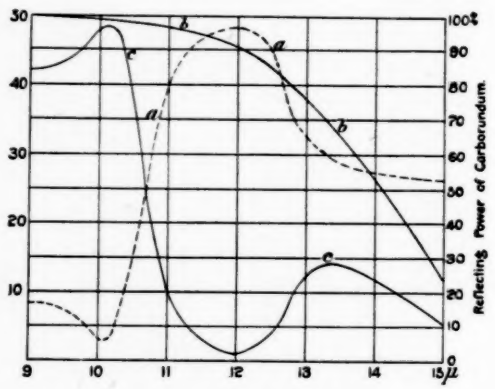

Fig. 4.

stance, viz., quartz, and in the latter it is the result of two well defined bands, at 8.5 and 9.03 μ. The effect of such a band on the dispersion of this substance must be very marked. In fact the unusual dispersion in the visible spectrum, found by others, no doubt is greatly influenced by this band. In this figure are given the computed energy curve, *b*, for a perfect radiator (using Planck's

[1] Aschinass, Verh. d. Deutsch. Phys. Ges., 17, p. 101, 1898.

[2] Rosenthal, Ann. der. Phys. (3), 68, p. 791, 1899.

formula, any temperature might have been selected instead of 300° Abs.) and the computed energy curve, *c*, of carborundum at the same temperature, using the formula $\varphi_\lambda = E_\lambda(1 - R_\lambda)$ where R is the observed reflecting power and E is the corresponding intensity of the complete radiator. It will be noticed that the radiation is highly selective, and that at 10 μ it approaches closely to that of the full radiator. If then one were to observe the emission curve of carborundum, the minimum at 10 μ would appear as an "emission band," but its explanation is different from that of emission bands of hot vapors, *e. g.*, CO_2 and H_2O in the Bunsen flame. Melted gold and copper are good examples of selective emission.

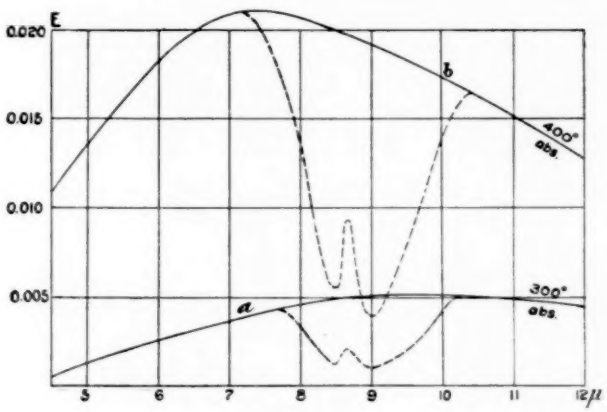

Fig. 5.

In Fig. 5 are given the computed emission curves, *a* and *b*, for a full radiator, for temperatures 300° and 400° Abs. respectively, and the corresponding emission curves [1] (dotted curves) of quartz, at the same temperature using the values of the reflecting power of quartz given by Rosenthal (loc. cit.). These values are somewhat lower than found by the writer. In general, the difference in the emissivity of the silicates and that of a full radiator, at 8 to 10 μ would not be so marked as in quartz. For mica (see Fig. 2) the emission minima would occur at 9.1 and 9.8 μ while for granite the emission minimum would extend from 8.5 to 10 μ. The effect of the combination would produce an appreciable depression in the radiation curve of a surface like that of the moon. Even, if we exclude

[1] Theoretically, of course, the dotted curves cannot touch the curve for a complete radiator, as here given.

atmospheric absorption, the emission curve will not be smooth and continuous, as some writers seem to think. If we superpose atmospheric absorption, the emission curve will be somewhat as shown in curve *b*, in Fig. 6. The transmission curve, *c*, is for a column of air 110 meters in length, containing 1.2 mm. precipitable water, and is due to Langley (loc. cit.). A thicker layer would shift the maximum at 8 μ toward 8.3 μ, found in the moon's radiation

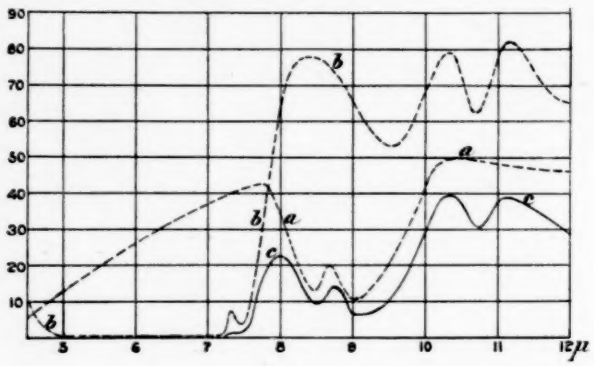

Fig. 6.

curve. In Fig. 7, curves *b*, *c*, *d*, show several of Langley's observed lunar radiation curves, and as a whole there is a close parallelism between the theoretical and the observed curve, especially at 10.7 μ where we have to consider only atmospheric absorption. From 8 to 9 μ, however, we have to consider the combined effect of atmosphere absorption, of incomplete emission of the moon (emission minima from 8.5 to 9.8 μ) and of selectively reflected energy from the sun. The latter will, no doubt, vary the most. The computed emission curve is the most intense at 10.2 μ while the observed curve is the most intense at 8.3 μ (see Fig. 7). This is to be expected if the observed energy curve is the composite of the selectively emitted energy of the moon, and the selectively reflected energy of the sun. The selectively reflected energy of the sun would, to a certain extent, fill up the minima in the lunar emission curve, and as far as our present knowledge goes would explain the observed curves, *b*, *c*, *d*, (Fig. 7), which lack a minimum at 8.5 μ. As a whole, from whatever standpoint we view this matter, we come to the same conclusion, viz.: that in the region from 8 to 10 μ the energy emitted from the moon consists of its own proper radiation and of reflected energy from the sun. This discussion of the de-

parture of the emissive power of the moon from that of a complete radiator applies equally well to the sun, which also does not radiate[1] like an "absolutely black body." Here the photospheric radiation is made up of the emission of particles at various depths and having a wide range of temperature. It is not surprising then,[2] that

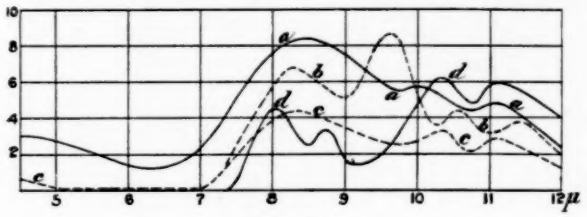

Fig. 7.

the radiation from the umbra of sun spots appears to be as intense as the still brilliant edge of the sun's disk. For, as far as our knowledge goes, the dark spots may be due to an inward convection[3] of photospheric material, thus forming an "ideal black body," the like of which, in magnitude, can not be duplicated in a laboratory.

If the brighter portions of the sun are composed of material having a higher reflecting power they may be at a higher temperature than the darker spots, and yet their emissivity may not be greater than in the latter, as in the cases just discussed.

To sum up, as matters now stand, we know that Langley observed radiation from the moon in the region of 8 to 10 μ. He observed also direct radiation from the sun in this region, but we do not know how much of this is superposed upon the direct radiation from the moon due to the latter being, without doubt, selectively reflecting in this region of the spectrum. As mentioned in a previous communication (loc. cit., p. 115), computations which require all sorts of assumptions will not settle the question. Bolometric comparison of the spectrum energy curves of the sun and of the moon made at high altitudes will be of greater service in clear-

[1] Very, Monthly Weather Review, No. 254, p. 22, 1901.

[2] Abbot, Astrophys. Journ., 23. p. 275, 1906.

[3] Whether there is a difference in the emissivity of the bright and dark spots appears to be an open question. An inrush in one region means an outrush in another region. An outrush of a transparent gas, if of sufficient depth might radiate like a "black body" and could have the same emissivity as a more opaque vapor which is at a higher temperature. It is of course unnecessary to assume convection to account for the equal emissivity of two bodies which are of different brightness optically. The difference in the emissivity of a full radiator and of platinum is sufficient to demonstrate this point.

ing up the matter. Very's[1] suggestion of searching for a solar image, in the lunar image, with a delicate heat-measuring instrument, covered by a screen with a pinhole aperture also deserves a thorough trial.

Exception has been taken to the writer's statement (loc. cit.) that, of the radiation observed by Langley in the lunar spectrum at 8 to 10 μ, we do not know how much is selectively reflected energy from the sun. But the writer maintains that just as soon as we admit the possibility of the moon being selectively reflecting in the region where it has its own proper radiation, then the use of glass[2] as an absorbing screen for testing the quality of that radiation is inadmissible, although it does serve as a rough test of the reflected energy at 0.5 to 3 μ. For this purpose it has been serviceable, and when we consider the great difficulties under which such work must be carried on, and the numerous correction factors that must be introduced (which at all times may be larger than the one for selective reflection) the use of a glass screen is not objectionable.

Furthermore, we have noticed that the silicates reflect as a transparent substance in all regions of the spectrum considered, except from 8 to 10 μ where it reflects like a metal. This means that the reflected energy in all these regions except 8 to 10 μ will consist of two parts, viz., that reflected from the outer surface, and that due to internal reflection. From 8 to 10 μ there will be but very little if any energy due to internal reflection. Hence to use the reflected energy spectrum of the moon from 0.5 to 4 μ as a means of estimating the amount of reflected energy at 8 to 10 μ is not a fair test, and, as used (for want of better data) in the present paper, can only serve as an approximation.

Washington, D. C.,
January 15, 1907.

[1] Very, Astrophys. Journ., 24, p. 353, 1906.

[2] Various observers have compared the total radiation from the moon to that part which is transmitted by glass. Since glass is opaque beyond 4 μ it was assumed to absorb the proper radiation from the moon (which must lie in the region of 8 to 10 μ) while the part transmitted by glass was considered to be the reflected energy of the sun. Evidently, if there is selectively reflected energy of the sun beyond 4 μ, it will be superposed upon the direct radiation of the moon, and will be absorbed by the glass screen, thus giving too low values for the reflected energy.